ACCREDITATION SCHEME FOR LABORATORIES

Guidance Notes EL 001

Guidelines on the Evaluation and Expression of Measurement Uncertainty for Electrical Testing Field

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Basic Principles on Measurement Uncertainty

1. Evaluation of Uncertainty

The uncertainty of the result of a measurement generally consists of several components. They can be grouped into two categories according to the method used to estimate their numerical values:

- **Type A evaluation**
  Calculation of uncertainty is by statistical analysis through repetitive observations.

- **Type B evaluation**
  Calculation of uncertainty is by means other than statistical analysis.

2. Modeling the Measurement Process

- A measurand Y can be determined from N inputs quantities X_1, X_2, X_3 … X_N, through a function f:

$$Y = f(X_1, X_2, X_3 \ldots X_N)$$

- An estimate of Y, denoted by y, is obtained from x_1, x_2, x_3 … x_N, the estimates of the input quantities X_1, X_2, X_3 … X_N, through the same function f:

$$y = f(x_1, x_2, x_3 \ldots x_N)$$

- The uncertainty associated with the estimate y is obtained by appropriately combining the estimated standard deviation (or standard uncertainty) of each of the input estimate x_i.

3. Type A Evaluation of Standard Uncertainty

- The arithmetic mean for n independent observations:
- The standard deviation of the n independent observations:

$$\bar{q} = \frac{1}{n}\sum_{k=1}^{n} q_k$$

$$s(q_k) = \sqrt{\frac{1}{n-1}\sum_{k=1}^{n}(q_k - \bar{q})^2}$$

- The standard deviation of the mean (estimate the spread of the distribution of the means):

$$s(\bar{q}) = \frac{s(q_k)}{\sqrt{n}}$$

- For an input estimate x_i determined from n repeated observations, the Type A standard uncertainty $u(x_i)$, with degrees of freedom v is given by:

$$u(x_i) = s(\bar{q})$$

$$v_i = n - 1$$

- Note: the degree of freedom should always be given when Type A evaluation of an uncertainty component is reported.

4. Type B Evaluation of Standard Uncertainty

- Covert a quoted uncertainty to a standard uncertainty from the knowledge of the probability distribution of the uncertainty.

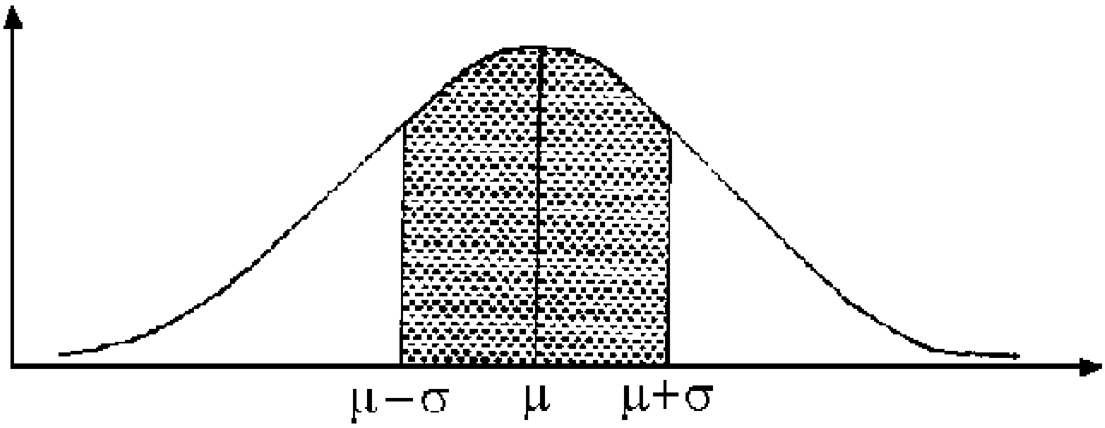
- Commonly used probability distributions:

  - Normal or Gaussian probability distribution

  - Rectangular probability distribution

- Degree of freedom is assumed to be infinite

Normal Probability Distribution

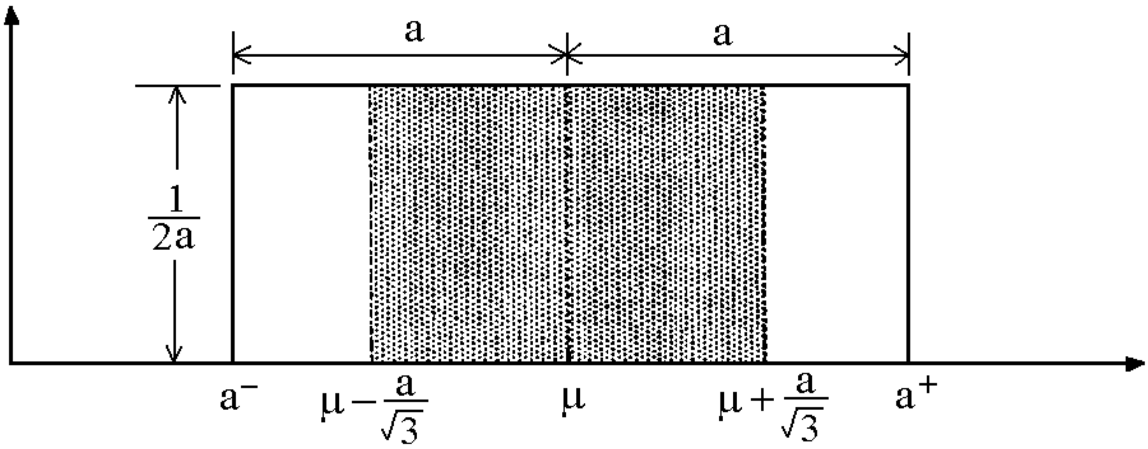


A normal distribution can be assumed when an uncertainty is quoted with a given confidence level. For example, a calibration report states that the uncertainty of a voltmeter is ± 0.1 V with a confidence level of 95%. The standard uncertainty of the voltmeter is given by:

$$u(x) = \sigma = \frac{k\sigma}{k} = \frac{0.1}{1.96} = 0.051\,\mathrm{V}$$

(Note: 95 % level of confidence has a coverage factor of 1.96)

Rectangular Probability Distribution



When an uncertainty is given by maximum bound within which all values are equally probable, the rectangular distribution can be assumed. For example, the accuracy of a voltmeter of a specific range is quoted as ± 0.2 V. The standard uncertainty of the voltmeter is given by:

$$u(x) = \frac{a}{\sqrt{3}} = \frac{0.2}{\sqrt{3}} = 0.115\,\text{V}$$

5. Combined Standard Uncertainty

The estimate of a measurand *Y* is given by:

$$y = f(x_1, x_2, x_3, \ldots\, x_N)$$

$$\Delta y = \frac{\partial f}{\partial x_1}\Delta x_1 + \frac{\partial f}{\partial x_2}\Delta x_2 + \frac{\partial f}{\partial x_3}\Delta x_3 + \Lambda \; + \frac{\partial f}{\partial x_N}\Delta x_N$$

It can be shown that the above equation leads to:

$$u_c^{\,2}(y) = \left(\frac{\partial f}{\partial x_1}\right)^2 u^2(x_1) + \left(\frac{\partial f}{\partial x_2}\right)^2 u^2(x_2) + \left(\frac{\partial f}{\partial x_3}\right)^2 u^2(x_3) + \Lambda \; + \left(\frac{\partial f}{\partial x_N}\right)^2 u^2(x_N)$$
$$= c_1^{\,2}u^2(x_1) + c_2^{\,2}u^2(x_2) + c_3^{\,2}u^2(x_3) + \Lambda \; + c_N^{\,2}u^2(x_N)$$

The combined standard uncertainty:

$$u_c(y) = \sqrt{c_1^{\,2}u^2(x_1) + c_2^{\,2}u^2(x_2) + c_3^{\,2}u^2(x_3) + \Lambda \; + c_N^{\,2}u^2(x_N)}$$

where c_1, c_2, $c_3 \ldots c_N$ are the sensitivity coefficients

Each component of the combined standard uncertainty could be calculated using either Type A or Type B evaluation method.

6. Coverage Factor of Combined Uncertainty

To determine the coverage factor of combined uncertainty, the effective degree of freedom must be first calculated from the *Welch-Satterthwaite* formula:

$$\nu_{eff} = \frac{u_c^{\,4}(y)}{\sum_{i=1}^{N} \frac{c_i^{\,4} u^4(x_i)}{\nu_i}}$$

Based on the calculated ν_{eff}, obtain the *t*-factor $t_p(\nu_{eff})$ for the required level of confidence *p* from the *t*-distribution table.

The coverage factor will be:

$$k_p = t_p(\nu_{eff})$$

7. Expanded Uncertainty

The expanded uncertainty defines an interval about the estimated result *y* within which the true value of the measurand *Y* is confidently believed to lie. It is given by:

$$U = k_p \, u_c(y)$$

The measurand *Y* is reported in the following format:

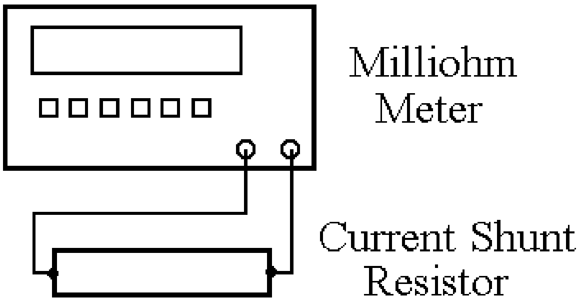
$$Y = y \pm U$$

It means that the true value of measurand *Y* is confidently believed to fall within the following range:

$$y - U \le Y \le y + U$$

Example #1: Resistance Measurement

A milliohm meter is used to measure the resistance of a current shunt resistor. At the selected range of the meter for the measurement, the calibration certificate states an uncertainty of ± 0.2 mΩ at 95 % of confidence level. Effects of room temperature and humidity on the measurement are found to be negligible.



Measurement record:

Reading	1	2	3	4	5	6	7	8	9	10
***R* (mΩ)**	9.4	9.1	9.4	9.8	9.7	9.4	9.8	9.7	9.4	9.4

1. Measurement Process Model

The measured resistance is given by:

$$R_x = R_{rdg} + \Delta R_m$$

where R_{rdg}: resistance reading recorded by the meter
ΔR_m: meter uncertainty

2. Uncertainty Equation

The combined standard uncertainty is given by:

$$u_c(R) = \sqrt{c_1^2 u^2(R_{rdg}) + c_2^2 u^2(\Delta R_m)}$$

Since $c_1 = \frac{\partial R_x}{\partial R_{rdg}} = 1$ and $c_2 = \frac{\partial R_x}{\partial(\Delta R_m)} = 1$, the combined standard uncertainty is give by:

$$u_c(R) = \sqrt{u^2(R_{rdg}) + u^2(\Delta R_m)}$$

where

$u(R_{rdg})$ is the standard uncertainty due to the repeatability of the meter reading

$u(\Delta R_m)$ is the standard uncertainty due to the meter calibration

3. Calculation of Uncertainty Components

<u>Type A evaluation:</u>
The best estimate of the measured resistance is given by the arithmetic mean:

$$\overline{R} = \frac{1}{10}\sum_{k=1}^{10} R_k = \frac{1}{10}(95.1) = 9.51\,\text{m}\Omega$$

Standard deviation:

$$s(R) = \sqrt{\frac{1}{10-1}\sum_{k=1}^{10}\left(R_k - \overline{R}\right)^2} = \sqrt{\frac{1}{9}(2.449)} = 0.522\,\text{m}\Omega$$

Standard uncertainty:

$$u\left(R_{rdg}\right) = s\left(\overline{R}\right) = \frac{s(R)}{\sqrt{n}} = \frac{0.522}{\sqrt{10}} = 0.165\,\text{m}\Omega$$

Degree of freedom, $\nu = 9$

<u>Type B evaluation:</u>
The uncertainty of the calibration is ± 0.2 mΩ with 95 % of confidence level (k = 1.96).

$$u\left(\Delta R_m\right) = \frac{0.2}{1.96} = 0.102\,\text{m}\Omega$$

Degree of freedom, $\nu = \infty$

Note: The value of 0.2 mΩ is used as a component for Type B evaluation on the assumption that the drift and stability of the equipment is negligible.

4. Uncertainty Budget Table

Source of Uncertainty	Type	Uncertainty Value (mΩ)	Probability Distribution	k	u_i (mΩ)	c_i	$c_i \times u_i$	ν_i
Repeatability $u(R_{rdg})$	A	0.165	-	-	0.165	1	0.165	9
Meter Calibration $u(\Delta R_m)$	B	0.200	Normal	1.96	0.102	1	0.102	∞

5. Combined Standard Uncertainty

$$u_c(R) = \sqrt{0.165^2 + 0.102^2} = 0.194\ \text{m}\Omega$$

6. Effective Degrees of Freedom

$$\nu_{eff} = \frac{0.194^4}{\dfrac{0.165^4}{9} + \dfrac{0.102^4}{\infty}} \approx 17$$

7. Expanded Uncertainty

For ν_{eff} = 17, the coverage factor of the combined standard uncertainty (k_p) is equal to 2.11 at 95 % level of confidence.

$$U = k_p \times u_c = 2.11 \times 0.194 = 0.409\ \text{m}\Omega$$

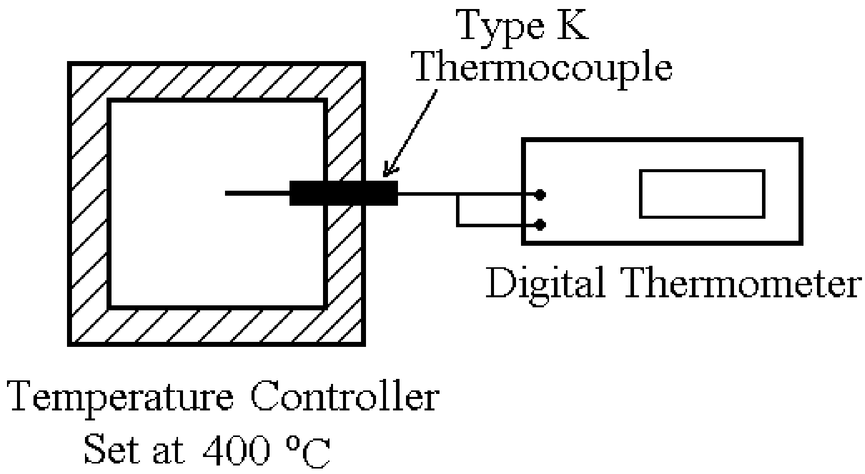
8. Reporting of Result

$$R = 9.51 \pm 0.409\ \text{m}\Omega$$

The measured resistance of the current shunt resistor is 9.51 mΩ. The expanded uncertainty is ± 0.409 mΩ with a coverage factor of 2.11, assuming a normal distribution at a level of confidence of 95 %.

Example #2: Temperature Measurement

A digital thermometer with a Type K thermocouple is used to measure the temperature inside a temperature chamber. The temperature controller of the chamber is set at 400°C.



Digital thermometer specification:

- Accuracy = ± 0.6 °C

Thermocouple specifications:

- Temperature correction for the thermocouple at 400 °C is 0.5 ± 1.0 °C at 95 % confidence level
- Deviation due to immersion = ± 0.1 °C
- Deviation due to drift = ± 0.2 °C

Measurement record:

S/N	1	2	3	4	5	6	7	8	9	10
T (°C)	400.1	400.0	400.1	399.9	399.9	400.0	400.1	400.2	400.0	399.9

1. Measurement Process Model

The measured temperature is given by:

$$t_x = t_{rdg} + \Delta t_m + \Delta t_{tc} + \Delta t_{imm} + \Delta t_{drift}$$

where

t_{rdg} is the temperature reading recoded by the digital thermometer
Δt_m is the accuracy of digital thermometer
Δt_{tc} is the temperature correction of the thermocouple

Δt_{imm} is the deviation due to immersion of the thermocouple
Δt_{drift} is the deviation due to drift of the thermocouple

2. Uncertainty Equation

$$u_c(t_x) = \sqrt{u^2(t_{rdg}) + u^2(\Delta t_m) + u^2(\Delta t_{tc}) + u^2(\Delta t_{imm}) + u^2(\Delta t_{drift})}$$

All the sensitivity coefficients are equal to unity.

3. Calculation of Uncertainty Components

<u>Type A evaluation:</u>
The best estimate of the measured temperature is given by the arithmetic mean:

$$\bar{T} = \frac{1}{10}\sum_{k=1}^{10} T_k = 400.02\ {}^{\circ}\mathrm{C}$$

Standard deviation:

$$s(T) = \sqrt{\frac{1}{10-1}\sum_{k=1}^{10}\left(T_k - \bar{T}\right)^2} = 0.103\ {}^{\circ}\mathrm{C}$$

Standard uncertainty:

$$u(t_{rdg}) = s(\bar{T}) = \frac{s(T)}{\sqrt{n}} = \frac{0.103}{\sqrt{10}} = 0.033\ {}^{\circ}\mathrm{C}$$

Degree of freedom, ν = 9

<u>Type B evaluation:</u>
The accuracy of the digital thermometer = ± 0.6 °C. Assume rectangular distribution, the standard uncertainty of the digital thermometer meter:

$$u(\Delta t_{dev}) = \frac{0.6}{\sqrt{3}} = 0.346\ {}^{\circ}\mathrm{C}$$

Degree of freedom, $\nu = \infty$

The uncertainty of the temperature correction of the thermocouple = ± 1.0 °C at 95 % confidence level (*k* =1.96). The standard uncertainty due to temperature correction:

$$u(\Delta t_{tc}) = \frac{1.0}{1.96} = 0.510\ {}^{\circ}\mathrm{C}$$

Degree of freedom, $\nu = \infty$

The uncertainty of the thermocouple due to immersion = ± 0.1 ºC. Assume rectangular distribution, the standard uncertainty due to immersion:

$$u(\Delta t_{imm}) = \frac{0.1}{\sqrt{3}} = 0.058 \ ^{\circ}\text{C}$$

Degree of freedom, $\nu = \infty$

The uncertainty of the thermocouple due to drift = ± 0.2 ºC. Assume rectangular distribution, the standard uncertainty due to drift:

$$u(\Delta t_{drift}) = \frac{0.2}{\sqrt{3}} = 0.115 \ ^{\circ}\text{C}$$

Degree of freedom, $\nu = \infty$

4. Uncertainty Budget Table

Source of Uncertainty	**Type**	**Uncertainty Value (ºC)**	**Probability Distribution**	***k***	**u_i (ºC)**	**c_i**	**$c_i \times u_i$**	**ν_i**
Repeatability $u(t_{rdg})$	A	0.033	-	-	0.033	1	0.033	9
Digital Thermometer $u(\Delta t_m)$	B	0.6	Rectangular	1.732	0.346	1	0.346	∞
Temperature correction $u(\Delta t_{tc})$	B	1.0	Normal	1.96	0.510	1	0.510	∞
Immersion $u(\Delta t_{imm})$	B	0.1	Rectangular	1.732	0.058	1	0.058	∞
Drift $u(\Delta t_{drift})$	B	0.2	Rectangular	1.732	0.115	1	0.115	∞

5. Combined Standard Uncertainty

$$u_c(t_x) = \sqrt{0.033^2 + 0.346^2 + 0.510^2 + 0.058^2 + 0.115^2} = 0.63 \ ^{\circ}\text{C}$$

6. Effective degrees of freedom

$$\begin{aligned} \nu_{eff} &= \frac{0.63^4}{\frac{0.033^4}{9} + \frac{0.510^4}{\infty} + \frac{0.058^4}{\infty} + \frac{0.115^4}{\infty} + \frac{0.346^4}{\infty} + \frac{0.029^4}{\infty}} \\ &= 1{,}195{,}498 \\ &\approx \infty \end{aligned}$$

7. Expanded Uncertainty

Degree of freedom for the combined standard uncertainty approaches ∞. Therefore, coverage factor of the combined standard uncertainty (k_p) is equal to 1.96 at 95 % level of confidence.

$$U = k_p \times u_c = 1.96 \times 0.63 = 1.235\ ^{o}C$$

8. Reporting of result

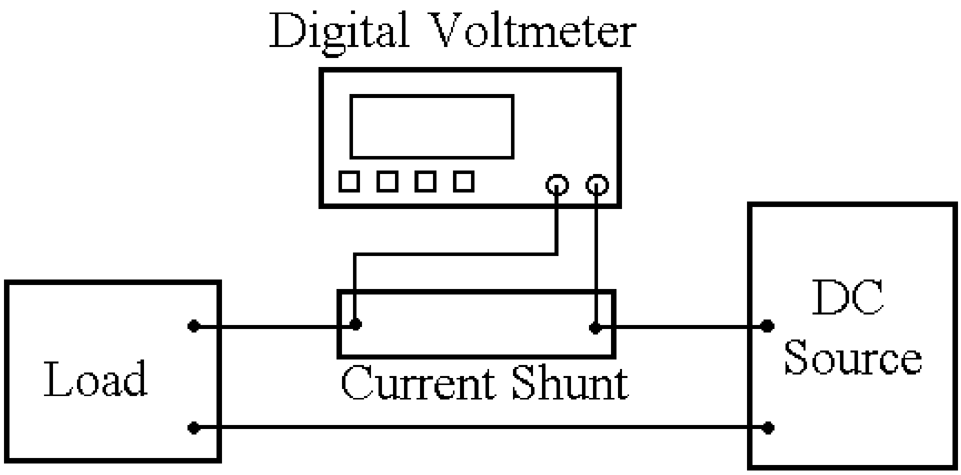
The correction at 400 ^{o}C is 0.5 ^{o}C, hence

$$T = (400.02 + 0.5) \pm 1.235\ ^{o}C = 400.52 \pm 1.235\ ^{o}C$$

The measured temperature of the chamber is 400.52 ^{o}C. The expanded uncertainty is $\pm$ 1.235 ^{o}C with a coverage factor of 1.96, assuming a normal distribution at a level of confidence of 95 %.

Example #3: Current Measurement

A current of 10 A is measured by using a current shunt and a voltmeter.



Current shunt specifications:

- The calibration report gives R = 0.010088 Ω at 10 A (23 °C) and expanded uncertainty = ± 0.08% at 95 % confidence level
- Temperature coefficient between 15 to 30 °C = 60 ppm/K
- Uncertainty due to resistance drift is negligible

Digital voltmeter specifications:

Under the condition of 15 to 40 °C

Range	Full scale	Uncertainty ±(% of reading + number of counts)
200 mV	199.99 mV	0.03 +2

Measurement record:
Room temperature = 23 ± 5 °C

Reading	1	2	3	4	5	6	7	8	9	10
Voltage (mV)	100.68	100.83	100.79	100.64	100.63	100.94	100.60	100.68	100.76	100.65

1. Measurement Process Model

$$I = f(V, R) = \frac{V}{R}$$

2. Uncertainty Equation

$$u_c^2(I) = \left(\frac{\partial I}{\partial V}\right)^2 u_1^2(V) + \left(\frac{\partial I}{\partial V}\right)^2 u_2^2(V) + \left(\frac{\partial I}{\partial R}\right)^2 u_3^2(R) + \left(\frac{\partial I}{\partial R}\right)^2 u_4^2(R)$$
$$= c_1^2\left[u_1^2(V) + u_2^2(V)\right] + c_2^2\left[u_3^2(R) + u_4^2(R)\right]$$

The sensitivity coefficients:

$$c_1 = \frac{\partial I}{\partial V} = \frac{1}{R} \quad \text{and} \quad c_1 = \frac{\partial I}{\partial R} = -\frac{V}{R^2}$$

where

$u_1(V)$: standard uncertainty of measured voltage due to repeatability
$u_2(V)$: standard uncertainty of measured voltage due to voltmeter resolution
$u_3(R)$: standard uncertainty of current shunt calibrated resistance value
$u_4(R)$: standard uncertainty of current shunt resistance due to temperature effect

3. Calculation of Uncertainty Components

Type A evaluation:

The best estimate of the measured voltage is given by the arithmetic mean:

$$\bar{V} = \frac{1}{10}\sum_{k=1}^{10} V_k = \frac{1}{10}(1007.2) = 100.72 \text{ mV}$$

Standard deviation:

$$s(V) = \sqrt{\frac{1}{10-1}\sum_{k=1}^{10}\left(V_k - \bar{V}\right)^2} = \sqrt{\frac{1}{9}\left(1040\times10^{-4}\right)} = 10.75\times10^{-2} \text{ mV}$$

Standard uncertainty:

$$u_1(V) = s(\bar{V}) = \frac{s(V)}{\sqrt{n}} = \frac{10.75\times10^{-2}}{\sqrt{10}} = 3.40\times10^{-2} \text{ mV}$$

Degree of freedom, ν_1 = 9

Type B evaluation:

The resolution of the voltmeter = ± 0.03 % of reading + 2 counts
= ± (0.03/100) × 100.72 + 2(0.01)
= ± 5.02 × 10^{-2} mV

Assuming rectangular distribution, the standard uncertainty due to voltmeter resolution:

$$u_2(V) = \frac{5.02\times10^{-2}}{\sqrt{3}} = 2.90\times10^{-3}\ \text{mV}$$

Degree of freedom, $\nu_2 = \infty$

The uncertainty of the shunt resistance = 0.08 % × 0.010088
=(0.08/100) × 0.010088
= 8.07 × 10^{-6} Ω

Normal distribution with 95 % level of confidence (k = 1.96)

$$u_3(R) = \frac{8.07\times10^{-6}}{1.96} = 4.12\times10^{-6}\ \Omega$$

Degree of freedom, $\nu_3 = \infty$

The uncertainty of the shunt resistance due to temperature effect:

$60 \times 10^{-6} \times \Delta t \times R = 60 \times 10^{-6} \times 5 \times 0.010088 = 3.03 \times 10^{-6}\ \Omega$

Assuming rectangular distribution,

$$u_4(R) = \frac{3.03\times10^{-6}}{\sqrt{3}} = 1.75\times10^{-6}\ \Omega$$

Degree of freedom, $\nu_4 = \infty$

$$c_1 = \frac{1}{R} = \frac{1}{0.010088} = 99.128\ \text{S}$$

$$c_1 = -\frac{V}{R^2} = -\frac{100.72\times10^{-3}}{0.010088^2} = -989.70\ \text{V}/\Omega^2$$

4. Uncertainty Budget Table

Source of Uncertainty	Type	Uncertainty Value	Probability Distribution	k	u_i	c_i	$c_i \times u_i$ (A)	v_i
Voltmeter Repeatability $u_1(V)$	A	3.40 × 10^{-2} mV	-	-	3.40 × 10^{-2} mV	99.128 S	3.37 × 10^{-3}	9
Voltmeter Resolution $u_2(V)$	B	5.02 × 10^{-2} mV	Rectangular	1.732	2.90 × 10^{-2} mV	99.128 S	2.87 × 10^{-2}	∞
Shunt Resistance $u_3(R)$	B	8.07 × 10^{-6} Ω	Normal	2	4.12 × 10^{-6} Ω	989.7 V/Ω²	4.08 × 10^{-3}	∞
Shunt Temp. Effect $u_4(R)$	B	3.03 × 10^{-6} Ω	Rectangular	1.732	1.75 × 10^{-6} Ω	989.7 V/Ω²	1.73× 10^{-3}	∞

5. Combined Standard Uncertainty

$$u_c^2(I) = c_1^2 u_1^2(V) + c_1^2 u_2^2(V) + c_2^2 u_3^2(R) + c_2^2 u_4^2(R)$$
$$= \left(3.37\times10^{-3}\right)^2 + \left(2.87\times10^{-3}\right)^2 + \left(4.08\times10^{-3}\right)^2 + \left(1.73\times10^{-3}\right)^2$$

$$u_c(I) = \sqrt{3.92\times10^{-5}} = 6.26\times10^{-3}\ \text{A}$$

6. Effective Degrees of Freedom

$$\nu_{eff} = \frac{\left(6.26\times10^{-3}\right)^4}{\frac{\left(3.37\times10^{-3}\right)^4}{9} + \frac{\left(2.87\times10^{-3}\right)^4}{\infty} + \frac{\left(4.08\times10^{-3}\right)^4}{\infty} + \frac{\left(1.73\times10^{-3}\right)^4}{\infty}}$$
$$= 107$$

7. Expanded Uncertainty

Since ν_{eff} = 10& >100, the coverage factor of the combined standard uncertainty (k_p) approaches 1.96 at 95 % level of confidence.

$$U = k_p \times u_c = 1.96 \times 6.26 \times 10^{-3} = 0.012\ \text{A}$$

8. Reporting of Results

$$\bar{I} = \frac{\bar{V}}{R} = \frac{100.72\times10^{-3}}{0.010088} = 9.984\ \text{A}$$

$$I = 9.984 \pm 0.012\ \text{A}$$

The measured current is 9.984 A. The expanded uncertainty is ± 0.012 A with a coverage factor of 1.96, assuming a normal distribution at a level of confidence of 95 %.

References:

1. SAC-SINGLAS Technical Guide 1: Guidelines on the Evaluation and Expression of Measurement Uncertainty, 2nd Edition, March 2001.

2. ISO Guide to the Expression of Uncertainty in Measurement, 1995.

3. NIST Technical Note 1297: Guidelines for Evaluating and Expressing the Uncertainty of NIST Measurement Results, 1994.