



ACCREDITATION SCHEME FOR LABORATORIES

Technical Guide 1
Guidelines on the Evaluation and
Expression of Measurement Uncertainty

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Technical Guide 1

Guidelines on the Evaluation and Expression of Measurement Uncertainty

Second Edition

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FOREWORD

ISO/IEC 17025: 1999: General requirements for the competence of testing and calibration laboratories, which replaces ISO/IEC Guide 25, requires all calibration and testing laboratories performing their own calibration to have a procedure to estimate the uncertainty of measurement for all calibrations and types of calibrations. SAC-SINGLAS has adopted this new standard as the criteria for accrediting competent testing and calibration laboratories.

In October 1998, a Working Group was convened to review and update the first edition of SINGLAS Technical Guide 1 "Guidelines on the evaluation and expression of the measurement uncertainty" published in July 1995, which was based on "Guide to the Expression of Uncertainty in Measurement, 1993(E)" or in short the GUM.

The second edition of the Technical Guide 1 was revised to align industry practice with the revised GUM (1995) and it contains more worked examples. These examples automatically supersede those in the first edition.

ISO/IEC Guide 25:1990: General Requirements for the Competence of Calibration and Testing Laboratories, required that "a statement of the estimated uncertainty of the calibration or test results" be reported in the calibration and test reports. In order to assist accredited calibration laboratories meet this requirement, in 1993, the SINGLAS Technical Committee on Calibration and Measurement decided that it was necessary to produce a simplified document for deriving measurement uncertainty.

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1.0 INTRODUCTION

- 1.1 Increasingly, many have recognized that correct evaluation of measurement uncertainty is integral to managing quality and costs in measurements. Uncertainty analysis leads to better understanding of the relative importance of various influence quantities on measurements.
- 1.2 This publication gives recommendations for the treatment of uncertainty contributions and is intended to be applicable to most, if not all, of SAC-SINGLAS accredited measurement results. This publication does not define or recommend what the uncertainty contributions are, or should be, since these are dependent on the nature of the calibration and the measurement process. Worked examples on the treatment of uncertainty are included for the temperature, electrical, mechanical, dimensional and optical areas.

2.0 CLASSIFICATION OF COMPONENTS OF UNCERTAINTY

- 2.1 The objective of a measurement is to determine the value of the measurand that is, the value of the particular quantity to be measured.
- 2.2 In general, the result of a measurement is only an approximation or estimate of the value of the measurand and thus is complete only when accompanied by a statement of the uncertainty of that estimate.
- 2.3 Traditionally, an error in the measurement result is viewed as having two components, namely, a random component and a systematic component. Random error presumably arises from unpredictable or stochastic temporal and spatial variations of influence quantities. Systematic error arises from a recognized effect of an influence quantity on a measurement result.
- 2.4 The uncertainty of the result of a measurement generally consists of several components which may be grouped into two types according to the method used to estimate their numerical values:
 - Type A - those evaluated by the statistical methods,
 - Type B - those evaluated by other means.
- 2.5 There is not always a simple correspondence between the classification of uncertainty components into categories A and B and the commonly used classification of uncertainty components as "random" and "systematic". The nature of an uncertainty component is conditioned by the use of the corresponding quantity, that is, how that quantity appears in the mathematical model that describes the measurement process. When the corresponding quantity is used in a different way, a "random" component may become a "systematic" component and vice versa. Thus the terms "random uncertainty" and "systematic uncertainty" can be misleading when generally applied. An alternative nomenclature that might be used is

"component of uncertainty arising from a random effect,"

"component of uncertainty arising from a systematic effect,"

where a random effect is one that gives rise to a possible random error in the current measurement process and a systematic effect is one that give rise to a possible systematic error in the current measurement process. In principle, an uncertainty component arising from a systematic effect may in some cases be evaluated by method A while in the other cases by method B, as may be an uncertainty component arising from a random effect.

3.0 MODELLING THE MEASUREMENT

3.1 The output measurand Y is usually determined from N input quantities, ie. X_1, X_2, \dots, X_N , through a function f :

$$Y = f(X_1, X_2, \Lambda X_N) \quad \dots(1)$$

3.2 The input quantities X_1, X_2, \dots, X_N may themselves be viewed as measurands and may themselves depend on other quantities, including corrections and correction factors for systematic effects, thereby leading to a complicated functional relationship f .

3.3 The function f contains every quantity that contribute a significant component of uncertainty to the measurement result - this includes all corrections and correction factors. If the function f does not model the measurement to the extent set by the required accuracy of the measurement result, additional input quantities must be included in f to eliminate the inadequacy.

3.4 An estimate of the measurand Y , denoted by y , is obtained from equation (1) using input estimates x_1, x_2, \dots, x_N , for the values of the N input quantities X_1, X_2, \dots, X_N . The output estimate y , which is the result of the measurement, is thus given by:

3.5 The estimated standard deviation associated with the output estimate y is obtained by

$$y = f(x_1, x_2, \Lambda x_N) \quad \dots(2)$$

appropriately combining the estimated standard deviation (termed as standard uncertainty and denoted by $u(x_i)$) of each input estimate x_i .

3.6 Each standard uncertainty $u(x_i)$ is obtained either from a Type A or Type B evaluation as described below. Type A evaluations of standard uncertainty components are founded on frequency distributions while Type B evaluations are founded on a priori distributions - in both cases these distributions are models that represent the current state of knowledge.

4.0 TYPE A EVALUATION OF STANDARD UNCERTAINTY

4.1 Type A evaluation of standard uncertainty is based on any valid statistical method in analysis of a series of observations.

4.2 A component of Type A evaluation of standard uncertainty arises from random effect. The Gaussian or Normal law of error forms the basis of the analytical study of random effects.

4.3 In most cases, the best available estimate of the expected value of a quantity q that varies randomly, is the arithmetic mean \bar{q} . The arithmetic mean for n independent observations is given by:

$$\bar{q} = \frac{1}{n} \sum_{k=1}^n q_k \quad \dots(3)$$

4.4 The experimental standard deviation $s(q_k)$ is used to estimate the distribution of q .

$$s(q_k) = \sqrt{\frac{1}{n-1} \sum_{k=1}^n (q_k - \bar{q})^2} \quad \dots(4)$$

4.5 The experimental standard deviation of the mean $s(\bar{q})$ is used to estimate the spread of the distribution of the means.

$$s(\bar{q}) = \frac{s(q_k)}{\sqrt{n}} \quad \dots(5)$$

4.6 One important factor deciding the number of readings required in a measurement process is the permissible precision error of the parameter. Averaging a set of readings is a common technique for reducing the random error of the result. However, the benefit gets progressively less as the number is increased and it is usually not necessary to make more than ten measurements.^[3]

4.7 For a quantity determined from n independent repeated observations, the Type A standard uncertainty u , with degrees of freedom ν is :

$$u(x_i) = s(\bar{q}) \quad \dots(6)$$

$$\nu_i = n - 1 \quad \dots(7)$$

4.8 The degrees of freedom ν should always be given when Type A evaluation of uncertainty components are documented.

4.9 It is sometimes commercially impractical to investigate the Type A standard uncertainty for every measurement. Where a practical type of measurement is performed frequently, provided there are no change in the measurement system or procedure, it may be sufficient to estimate an uncertainty in terms of an investigation carried out earlier, and verified at discrete intervals of time to ensure that the measurement has not been degraded.

4.10 For a well-characterized measurement under statistical control, a pooled experimental standard deviation s_p , with degrees of freedom ν_p , based on M series of observations of the same variable may be available. The pooled experimental standard deviation is determined by:

$$s_p^2 = \frac{\sum_{i=1}^M \nu_i s_i^2}{\sum_{i=1}^M \nu_i} \quad \dots(8)$$

$$\nu_p = \sum_{i=1}^M \nu_i \quad \dots(9)$$

where s_i is the experimental standard deviation from one series of m_i independent repeated observations, and has degrees of freedom of $\nu_i = m_i - 1$.

4.11 If a measurand \bar{q} of the same variable is determined from n independent observations, the Type A standard uncertainty u can be estimated by:

$$u(x_i) = \frac{s_p}{\sqrt{n}} \quad \dots(10)$$

with effective degrees of freedom = ν_p

4.12 If an identical test has not previously been carried out, it is sometimes possible to obtain acceptable estimates of uncertainty due to random effect by interpolation or extrapolation from well-known statistical methods such as linear regression or other appropriate investigations.

5.0 TYPE B EVALUATION OF STANDARD UNCERTAINTY

5.1 Type B evaluation of standard uncertainty is obtained by means other than the statistical analysis of a series of observations. It is usually based on scientific judgement using all relevant information available, which may include:

- previous measurement data
- experience with, or general knowledge of the behaviour and property of relevant materials and instruments
- manufacturer's specification
- data provided in calibration and other reports and
- uncertainties assigned to reference data taken from data book

5.2 When considering Type B uncertainty, we have to convert the quoted uncertainty to a standard uncertainty. Different cases of how this is done are given below.

5.3 Convert a quoted uncertainty that is a stated multiple of an estimate standard deviation to a standard uncertainty by dividing the quoted uncertainty by the multiplier.

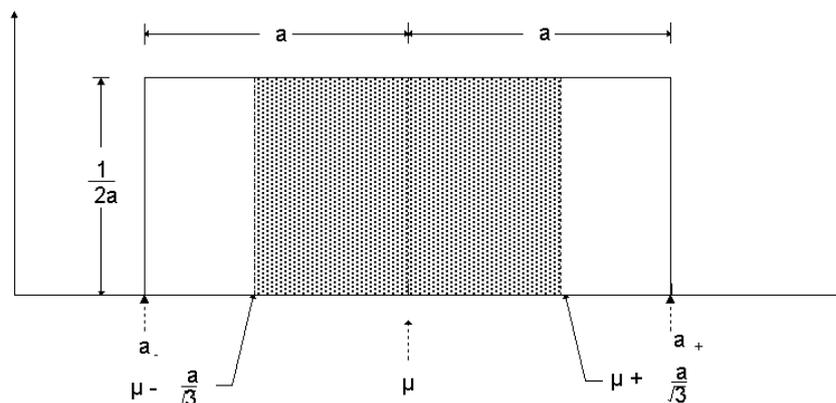
Example

A calibration report states that the measurement uncertainty is 3 mV at 3 standard deviations. The standard uncertainty is then 3 mV divided by 3 which gives 1 mV.

5.4 Convert a quoted uncertainty to a standard uncertainty from the knowledge of the probability distribution of the uncertainty. Divide the quoted uncertainty by a factor which depends on the probability distribution.

5.4.1 Rectangular Probability Distribution

It is used when uncertainties are given by maximum bound within which all values are equally probable. The standard uncertainty is computed by dividing the half-interval 'a' by $\sqrt{3}$.



Rectangular Distribution

Examples

a. The measurement accuracy of a voltmeter is $\pm 0.05\%$. The half-interval limit is 0.05% and the standard uncertainty is given by:

$$u(V) = \frac{0.05\%}{\sqrt{3}} \quad \dots(11)$$

- b. The resolution of a digital voltage display is 1 mV. Thus the interval is 1 mV and the half-interval limit is half of 1 mV. The standard uncertainty is then given by:

$$u(R) = \frac{0.5}{\sqrt{3}} \times 1 \text{ mV} \quad \dots(12)$$

- c. The hysteresis effect of an instrument is 0.1 %. The difference between maximum and minimum for the same input gives the interval and the half-interval limit is half of 0.1 %. The standard uncertainty is given by:

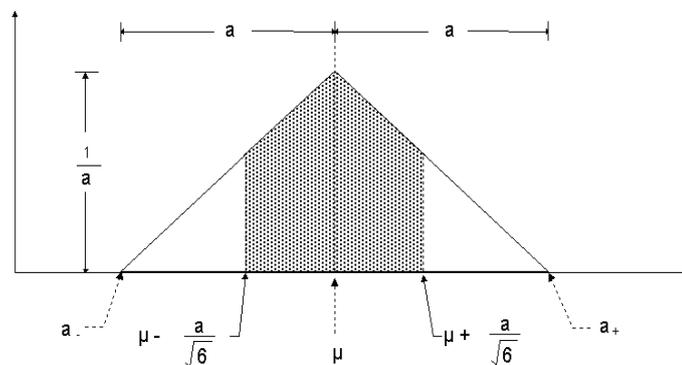
$$u(H) = \frac{0.5}{\sqrt{3}} \times 0.1 \% \quad \dots(13)$$

- d. The maximum drift of the value of a capacitance standard between calibration intervals is 0.001 pF. The history of the capacitance standard for the past few years showed that the capacitance value changed by not more than 0.001 pF. The standard uncertainty is then given by:

$$u(\text{drift}) = \frac{0.001 \text{ pF}}{\sqrt{3}} \quad \dots(14)$$

5.4.2 Triangular Probability Distribution

The triangular distribution is a better model if it is known that most of the values are likely to be near the centre of the distribution. The standard uncertainty is computed by dividing the half-interval 'a' by $\sqrt{6}$.



Triangular Distribution

Example

The environmental temperature is controlled such that it is always near the centre of the range 20 ± 2 °C. The half-interval limit is then 2 °C and the standard uncertainty is given by:

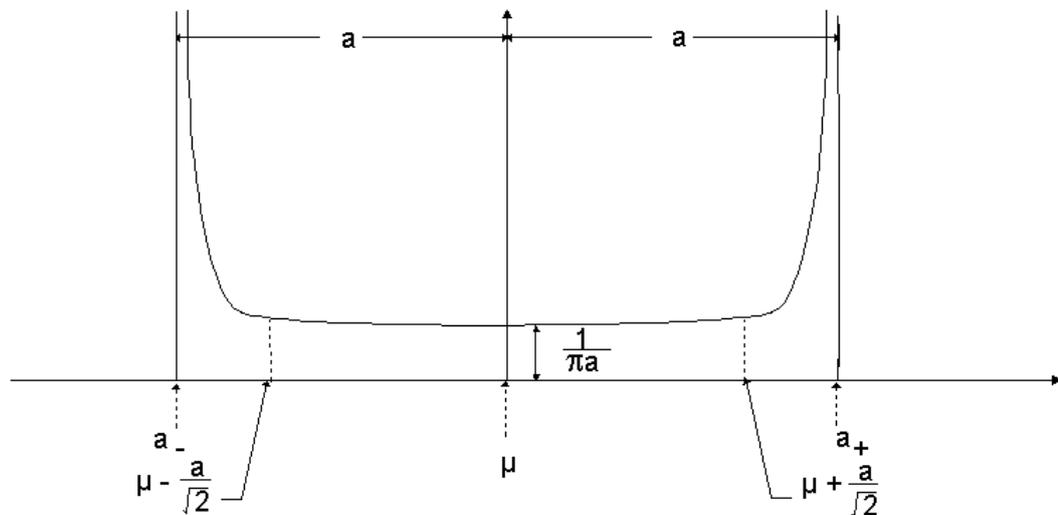
$$u(T) = \frac{2 \text{ °C}}{\sqrt{6}} \quad \dots(15)$$

5.4.3 U-Shape Probability Distribution

This distribution is used typically in the case of the mismatch uncertainty in radio and microwave power frequency power measurement. At high frequency, the power is delivered from a source to a load, and reflection occurs when the impedances do not match. The mismatch uncertainty is given by $2\Gamma_S\Gamma_L$.

Γ_S and Γ_L are the reflection coefficients of the source and the load respectively. The standard uncertainty is given by:

$$u(m) = \frac{2\Gamma_S\Gamma_L}{\sqrt{2}} \quad \dots(16)$$



U-shape Distribution

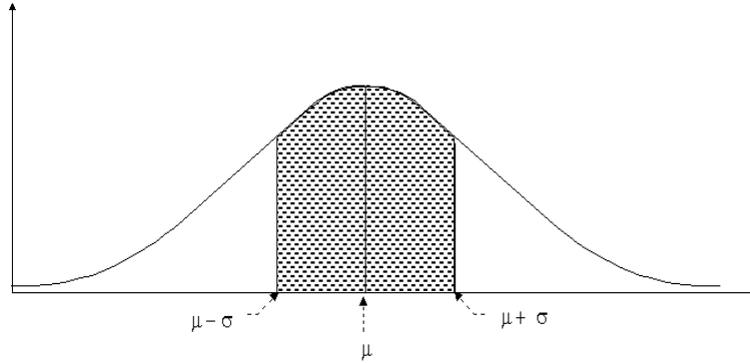
Example

The output power of a signal generator is measured by a power meter. The magnitude of the reflection coefficients of the signal generator and the power meter is 0.2 and 0.091 respectively. The standard uncertainty due to the mismatch is given by:

$$u(m) = \frac{2 \times 0.2 \times 0.091}{\sqrt{2}} \quad \dots(17)$$

5.4.4 Normal or Gaussian Probability Distribution

This distribution form can be assumed for an uncertainty that defines a confidence interval having a given level of confidence of say 95 % or 99 %. The standard uncertainty is obtained by dividing the quoted uncertainty by the appropriate factor for such a distribution (refer to APPENDIX 2 on the choice of factors).



Gaussian Distribution

Examples

- a. A calibration report states that the uncertainty is ± 0.1 dB with a coverage factor of 2.63. The standard uncertainty is given by:

$$u(x) = \frac{0.1 \text{ dB}}{2.63} \quad \dots(18)$$

- b. The uncertainty specification on a DC 1 V range is given as ± 5 ppm at a confidence level of 99 %. Assuming normally distributed, the standard uncertainty is given by:

$$u(x_1) = \frac{5 \text{ } \mu\text{V}}{2.58} \quad \dots(19)$$

5.5 The rectangular distribution is a reasonable default model in the absence of any other information. But if it is known that the value of the quantity in question is near the centre of the limits, a triangular or a normal distribution may be a better model.

5.6 Type B uncertainty is obtained from a priori probability distributions. It is implicitly assumed that the probability distribution is exactly known. In most cases, we can assume that the degrees of freedom for such standard uncertainty as infinite. This is a reasonable assumption as it is a common practice to choose a Type B uncertainty such that the probability of the quantity in question lying outside the uncertainty band is extremely small. (Refer to APPENDIX 1 for treatment of degrees of freedom).

6.0 COMBINED STANDARD UNCERTAINTY

- 6.1 The estimated standard deviation associated with the output estimate or measurement result y , termed combined standard uncertainty and denoted by $u_c(y)$, is determined from the standard uncertainty $u(x_i)$ associated with each input estimate x_i (refer to section 3 on 'Modelling the Measurement').
- 6.2 The following equations of computing the combined standard uncertainty are based on a first order Taylor series approximation of the equation (1). This method is often called the law of propagation of uncertainty.
- 6.3 When the non-linearity of f is significant, higher-order terms in the Taylor series expansion must be included in the expression for $u_c^2(y)$.

6.4 Uncorrelated Input Quantities

- 6.4.1 In the case where all input quantities are statistically independent, their joint probability distributions are products of their individual probability distributions.
- 6.4.2 The combined standard uncertainty $u_c(y)$ is the positive square root of the combined variance $u_c^2(y)$ which is given by:

$$u_c^2(y) = \sum_{i=1}^N \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) \quad \dots(20)$$

where f is the function given in equation (1)

- 6.4.3 The partial derivatives $\frac{\partial f}{\partial x_i}$ are called sensitivity coefficients. They describe how the output estimate y varies with changes in the values of the input estimates x_1, x_2, \dots, x_N . The combined variance $u_c^2(y)$ can be viewed as a sum of terms. These terms represent the estimated variance associated with the output estimate y generated by the estimated variance associated with each input estimate x_i .
- 6.4.4 Denoting $\frac{\partial f}{\partial x_i}$ by c_i , the equation (20) can be written as:

$$u_c^2(y) = \sum_{i=1}^N [c_i u(x_i)]^2$$

$$= \sum_{i=1}^N u_i^2(y) \quad \dots(21)$$

where: $u_i(y) = c_i u(x_i)$ and $c_i = \frac{\partial f}{\partial x_i}$

6.4.5 The sensitivity coefficients $\frac{\partial f}{\partial x_i}$ are sometimes determined experimentally. In this case, the knowledge of the function f is accordingly reduced to an empirical first-order Taylor series expansion based on the measured sensitivity coefficients.

6.4.6 If Y is of the form, ie. $Y = c X_1^{P_1} X_2^{P_2} \dots X_N^{P_N}$ with the exponents P_i known to be positive or negative numbers, the combined variance equation (20) can be expressed as:

$$\left[\frac{u_c(y)}{y} \right]^2 = \sum_{i=1}^N \left[\frac{P_i u(x_i)}{(x_i)} \right]^2 \quad \dots(22)$$

6.4.7 If each P_i is either +1 or -1, equation (22) becomes equation (23) which shows that for this special case the relative combined variance associated with the estimate y is simply equal to the sum of the estimated relative variances associated with the input estimates x_i .

$$\left[\frac{u_c(y)}{y} \right]^2 = \sum_{i=1}^N \left[\frac{u(x_i)}{(x_i)} \right]^2 \quad \dots(23)$$

6.5 Correlated Input Quantities

6.5.1 In the case where two or more input quantities are interdependent. The appropriate expression for the combined variance $u_c^2(y)$ associated with the result of a measurement is :

$$u_c^2(y) = \sum_{i=1}^N \sum_{j=1}^N \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(x_i, x_j) \quad \dots(24)$$

$$u_c^2(y) = \sum_{i=1}^N \left[\frac{\partial f}{\partial x_i} \right]^2 u^2(x_i) + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^N \frac{\partial f}{\partial x_i} \frac{\partial f}{\partial x_j} u(x_i, x_j) \quad \dots(25)$$

where $u(x_i, x_j)$ is the estimated covariance between x_i and x_j

6.5.2 The interdependence of two variables is characterized by their coefficients.

$$r(x_i, x_j) = \frac{u(x_i, x_j)}{u(x_i) u(x_j)} \quad \dots(26)$$

6.5.3 With the aid of equation (26) and the sensitivity coefficients, c_i described in the paragraph 6.4.3 and 6.4.4, the equation (25) becomes:

$$u_c^2(y) = \sum_{i=1}^N c_i^2 u^2(x_i) + 2 \sum_{i=1}^{N-1} \sum_{j=i+1}^N c_i c_j u(x_i) u(x_j) r(x_i, x_j) \quad \dots(27)$$

6.5.4 Correlation can occur if the same measurement is used more than once in the same measurement process, however, its effect on the combined uncertainty may be positive, i.e. the uncertainty is increased or negative which will lead to a reduction in the uncertainty.

6.5.5 If positive correlation is suspected but the correlation coefficient cannot be easily calculated then it is reasonable to assume a correlation coefficient of +1 and simply add the correlated uncertainty contributions.

6.5.6 If all of the input estimates are correlated with correlation coefficients $r(x_i, x_j) = +1$, equation (27) reduces to :

$$\begin{aligned} u_c^2(y) &= \left[\sum_{i=1}^N c_i u(x_i) \right]^2 \\ &= \left[\sum_{i=1}^N \frac{\partial f}{\partial x_i} u(x_i) \right]^2 \quad \dots(28) \end{aligned}$$

6.5.7 “In the case of two randomly varying quantities q and r , with their respective means \bar{q} and \bar{r} calculated from n independent pairs of simultaneous observations of q and r made under the same conditions of measurement, the covariance is estimated by:”

$$s(\bar{q}, \bar{r}) = \frac{1}{n(n-1)} \sum_{k=1}^n (q_k - \bar{q})(r_k - \bar{r}) \quad \dots(29)$$

where q , and r are the individual observations made simultaneously.

6.5.8 For two correlated input quantities X_i and X_j , the correlation coefficient is obtained from equation (26).

$$\begin{aligned} r(x_i, x_j) &= r(\bar{X}_i, \bar{X}_j) \\ &= \frac{s(\bar{X}_i, \bar{X}_j)}{s(\bar{X}_i) s(\bar{X}_j)} \end{aligned} \quad \dots(30)$$

7.0 EXPANDED UNCERTAINTY

7.1 While the combined standard uncertainty $u_c(y)$ can be used to express the uncertainty of measurement results, many applications required a measure of uncertainty that defines an interval about the measurement result y within which the value of the measurand Y can be confidently asserted to lie.

7.2 The measure of uncertainty intended to meet this requirement is termed expanded uncertainty, denoted by symbol U and is obtained by multiplying $u_c(y)$ by a coverage factor. Thus $U = k u_c(y)$ and it can be confidently asserted that $y - U \leq Y \leq y + U$, commonly written as $Y = y \pm U$.

Current international practice is to give a level of confidence of approximately 95 % (95.45 %). When level of confidence is fixed, k varies with effective degrees of freedom. Although in many cases, k equal to 2 can be used where effective degrees of freedom is reasonably large, greater or equal to 30. For cases where effective degrees of freedom is small, it is necessary to obtain the value of k using the t-distribution table. The calculation of effective degrees of freedom is outlined in Appendix 1.

7.3 In certain cases, whereby $u_c(y)$ is dominated by one component of uncertainty derived from Type B evaluation, the use of k value obtained from the t-distribution table is likely to give an interval corresponding to a level of confidence more than approximately 95 %. In this situation, knowledge of the assumed probability distribution of the measurand Y is required to obtain the value of k that produces an interval corresponding to a level of confidence of approximately 95 %. Refer to Annex G of Reference [1]

8.0 REPORTING OF RESULTS

8.1 In stating the measurement result in the calibration report, the following information shall be provided.

- a. the measurement result,
- b. the expanded uncertainty,
- c. the level of confidence used in defining the interval of the expanded uncertainty, and
- d. the coverage factor, k (and if necessary the effective degrees of freedom) used in defining the interval of the expanded uncertainty.

Example

The measurement result is $Y = y \pm U$. The reported expanded uncertainty of measurement is stated as the standard uncertainty of measurement multiplied by the coverage factor $k = 2$, which is for a level of confidence of approximately 95 %.

- 8.2 The numerical value of the uncertainty of measurement should be given to at most two significant figures. If the rounding brings the numerical value of the uncertainty of measurement down by more than 5 %, the rounding up value should be used.
- 8.3 The numerical value of the measurement result y should in the final statement normally be rounded to the least significant figure in the value of the expanded uncertainty assigned to the measurement result.
- 8.4 For the process of rounding, the usual rules for rounding of numbers based on ISO 31-0:1992, Annex B can be used. To round a number to n significant figures, discard all digits to the right of the n th place. If the discarded number is less than one-half a unit in the n th place, leave the n th digit unchanged. If the discarded number is greater than one-half a unit in the n th place, increase the n th digit by 1. If the discarded number is exactly one-half a unit in the n th place, leave the n th digit unchanged if it is an even number and add 1 to it if it is odd.

Examples of rounding a number to four significant figures.

20.453 mm 20.45 mm

20.456 mm 20.46 mm

20.455 mm 20.46 mm

20.465 mm 20.46 mm

9.0 CONCLUSION

- 9.1 When evaluating the measurement uncertainty, it is important not to 'double-count' uncertainty components. If a component of uncertainty arising from a particular effect is obtained from a Type B evaluation, it should be included as an independent component of uncertainty in the calculation of the combined standard uncertainty of the measurement result only to the extent that the effect does not contribute to the observed variability of the observations. This is because the uncertainty due to that portion of the effect that contributes to the observed variability is already included in the component of uncertainty obtained from statistical analysis of the observations.
- 9.2 Although this guide provides a framework for evaluating uncertainty, it cannot replace analytical thinking, skill and professionalism. The treatment of uncertainty is neither a routine task nor a purely mathematical one; it depends on detailed knowledge of the nature of the measurand and of the measurement system. The quality and usage of the measurement uncertainty quoted ultimately depends on the knowledge, critical investigation, and integrity of those who contribute to the assignment of the value.

10 REFERENCES

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**APPENDIX 1
COVERAGE FACTOR**

- 1 To estimate the value of such a coverage factor requires taking into account the uncertainty of $u_c(y)$, which is characterized by the "effective degrees of freedom," ν_{eff} , of $u_c(y)$. For example, if ν_{eff} is less than 11, simply assuming that the uncertainty of $u_c(y)$ is negligible and taking $k = 2$ may be inadequate if an expanded uncertainty $U = k u_c(y)$ that defines an interval having a level of confidence close to 95 % is required for a specific application. In this case, and in other similar cases where ν_{eff} of $u_c(y)$ is comparatively small and an interval having a level of confidence close to a specified level is required, it is unlikely that the uncertainty of $u_c(y)$ would be considered negligible. Instead, the small value of ν_{eff} , and thus the uncertainty of $u_c(y)$, would probably be taken into account when determining k .

- 2 The four-step procedure for calculating k is as follows :

- 1) Obtain y and $u_c(y)$.
- 2) Estimate the effective degrees of freedom ν_{eff} of $u_c(y)$ from the Welch-Satterthwaite formula:

$$\nu_{eff} = \frac{u_c^4(y)}{\sum_{i=1}^N \frac{c_i^4 u^4(x_i)}{\nu_i}}$$

where $c_i = \frac{\partial f}{\partial x_i}$, all of the $u(x_i)$ are mutually statistically independent, ν_i is the degrees of freedom of $u(x_i)$ and

$$\nu_{eff} \leq \sum_{i=1}^N \nu_i$$

The degrees of freedom of a standard uncertainty $u(x_i)$ obtained from Type A evaluation, where $x_i = X_i$ and $u(x_i) = s(X_i)$, is $\nu_i = n - 1$. If m parameters are estimated by fitting a curve to an n data points by the method of least squares, the degrees of freedom of the standard uncertainty of each parameter is $n - m$.

The degrees of freedom associated with a standard uncertainty $u(x_i)$, obtained from a Type B evaluation with lower and upper limits a_- and a_+ , are set in such a way that the probability of the quantity in question lying outside these limits is in fact extremely small, the degrees of freedom may be taken to be $\nu_i \rightarrow \infty$. Otherwise, the following equation may be used to define the degrees of freedom, ν , that is to be associated with a standard uncertainty $u(x_i)$ obtained from a Type B evaluation :

APPENDIX 1 COVERAGE FACTOR

$$\nu \approx \frac{1}{2} \left[\frac{u^2(x_i)}{\sigma^2 [u(x_i)]} \right]$$
$$\approx \frac{1}{2} \left[\frac{\Delta u(x_i)}{u(x_i)} \right]^{-2}$$

The quantity in large brackets is the relative uncertainty of $u(x_i)$, which for a Type B evaluation is a subjective quantity. This value is to be obtained from experience and knowledge of the measurement procedures.

Example :

Assume it is judged that the evaluation of $u(x_i)$ is reliable to about 25 %. Then it may be taken to mean that the relative uncertainty $\frac{\Delta u(x_i)}{u(x_i)} = 0.25$ and from the

above equation $\nu = \frac{0.25^{-2}}{2} = 8$

- 3) Obtain the t -factor $t_p(\nu_{eff})$ for the required level of confidence p from a table of values of $t_p(\nu)$ from the Student's t -distribution, (given in APPENDIX 2.) If ν_{eff} is not an integer, which will usually be the case, either interpolate or truncate ν_{eff} to the next lower integer.
- 4) Take $k = t_p(\nu_{eff})$ and calculate $U = k u_c(y)$.

APPENDIX 2

t-distribution

t-DISTRIBUTION

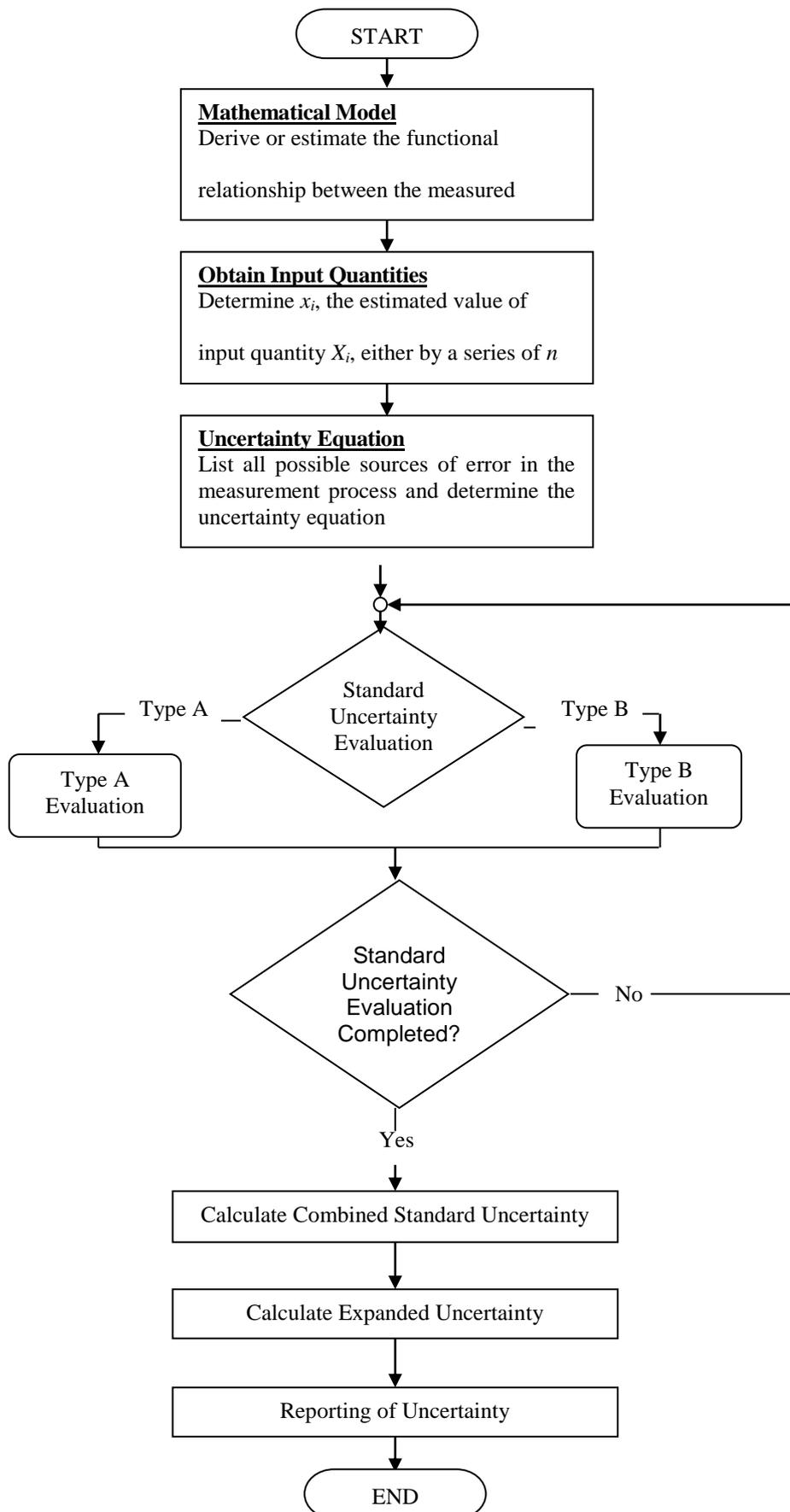
Value of $t_p(\nu)$ from the t -distribution for degrees of freedom ν that defines an interval $-t_p(\nu)$ to $+t_p(\nu)$ that encompasses the fraction p of the distribution

Degrees of freedom ν	Fraction p in percent					
	68.27 ^(a)	90.00	95.00	95.45 ^(a)	99.00	99.73 ^(a)
1	1.84	6.31	12.71	13.97	63.66	235.8
2	1.32	2.92	4.30	4.53	9.92	19.21
3	1.20	2.35	3.18	3.31	5.84	9.22
4	1.14	2.13	2.78	2.87	4.60	6.62
5	1.11	2.02	2.57	2.65	4.03	5.51
6	1.09	1.94	2.45	2.52	3.71	4.90
7	1.08	1.89	2.36	2.43	3.50	4.53
8	1.07	1.86	2.31	2.37	3.36	4.28
9	1.06	1.83	2.26	2.32	3.25	4.09
10	1.05	1.81	2.23	2.28	3.17	3.96
11	1.05	1.80	2.20	2.25	3.11	3.85
12	1.04	1.78	2.18	2.23	3.05	3.76
13	1.04	1.77	2.16	2.21	3.01	3.69
14	1.04	1.76	2.14	2.20	2.98	3.64
15	1.03	1.75	2.13	2.18	2.95	3.59
16	1.03	1.75	2.12	2.17	2.92	3.54
17	1.03	1.74	2.11	2.16	2.90	3.51
18	1.03	1.73	2.10	2.15	2.88	3.48
19	1.03	1.73	2.09	2.14	2.86	3.45
20	1.03	1.72	2.09	2.13	2.85	3.42
25	1.02	1.71	2.06	2.11	2.79	3.33
30	1.02	1.70	2.04	2.09	2.75	3.27
35	1.01	1.70	2.03	2.07	2.72	3.23
40	1.01	1.68	2.02	2.06	2.70	3.20
45	1.01	1.68	2.01	2.06	2.69	3.18
50	1.01	1.68	2.01	2.05	2.68	3.16
100	1.005	1.660	1.984	2.025	2.626	3.077
∞	1.000	1.645	1.960	2.000	2.576	3.000

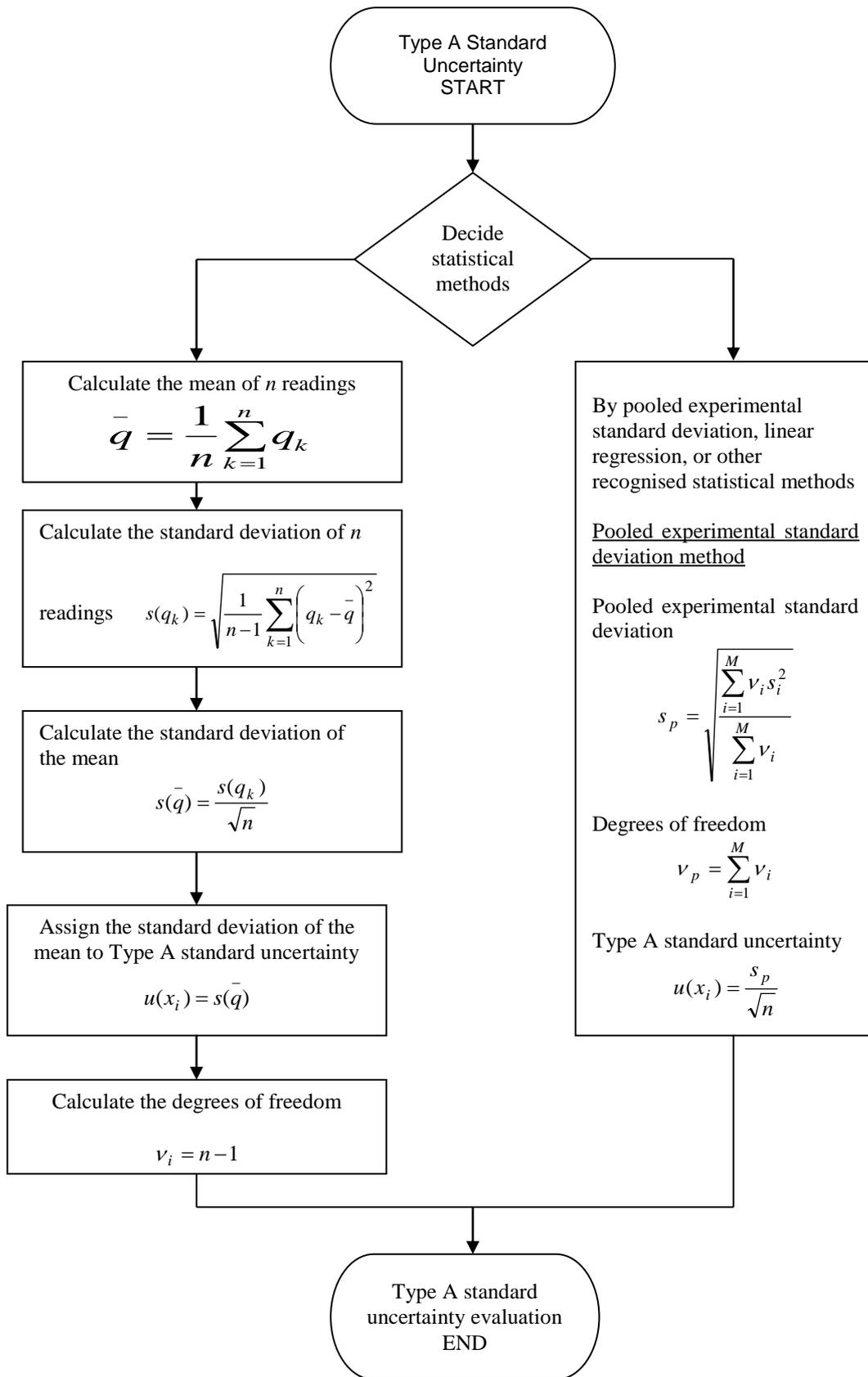
^(a)For a quantity z described by a normal distribution with expectation μ_z and standard deviation σ , the interval $\mu_z \pm k\sigma$ encompasses $p = 68.27, 95.45,$ and 99.73 percent of the distribution for $k = 1, 2$ and $3,$ respectively.

APPENDIX 3
FLOWCHART FOR EVALUATING UNCERTAINTY OF MEASUREMENT

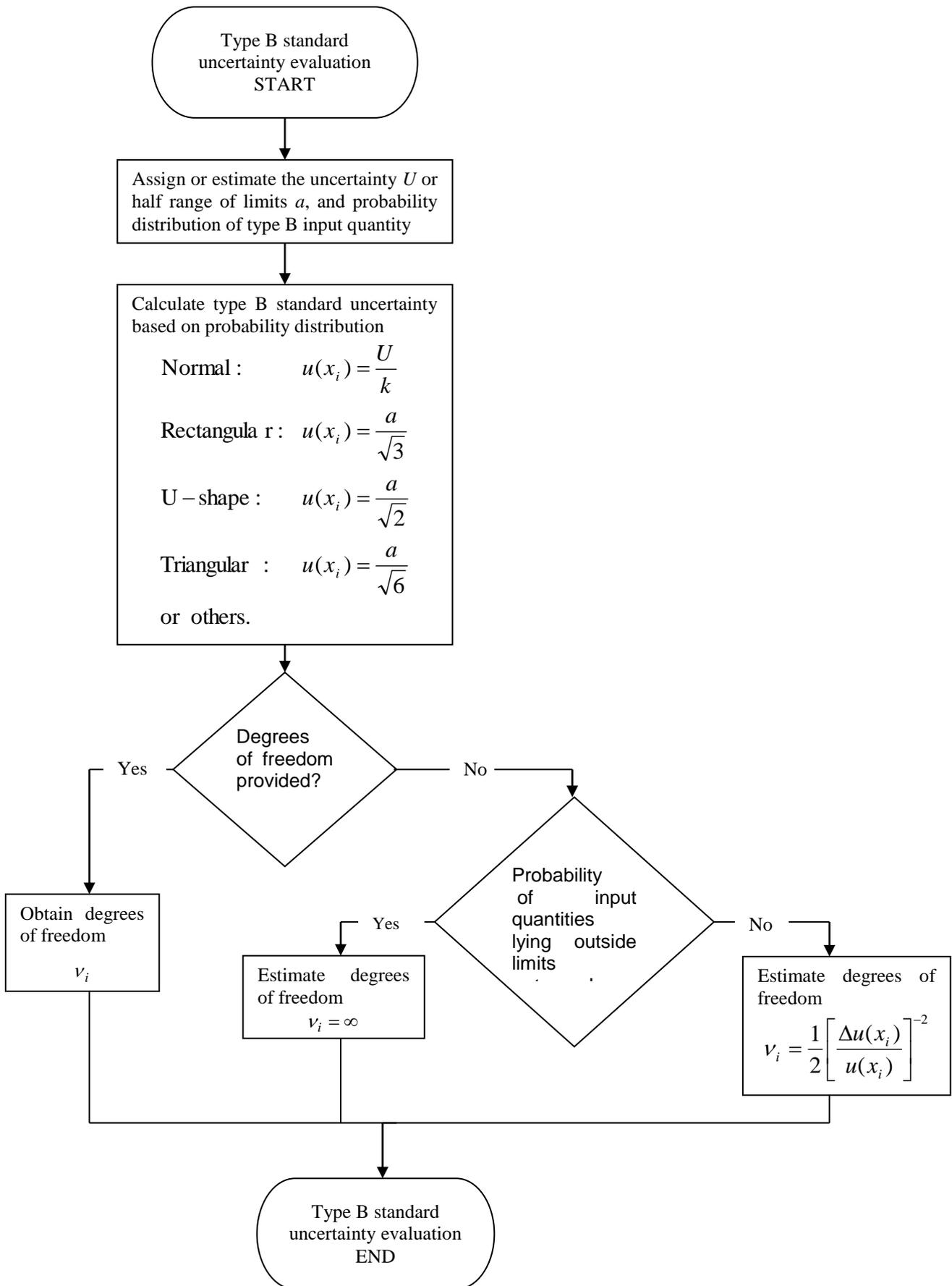
FLOWCHART OF SUMMARY FOR EVALUATING UNCERTAINTY OF MEASUREMENT



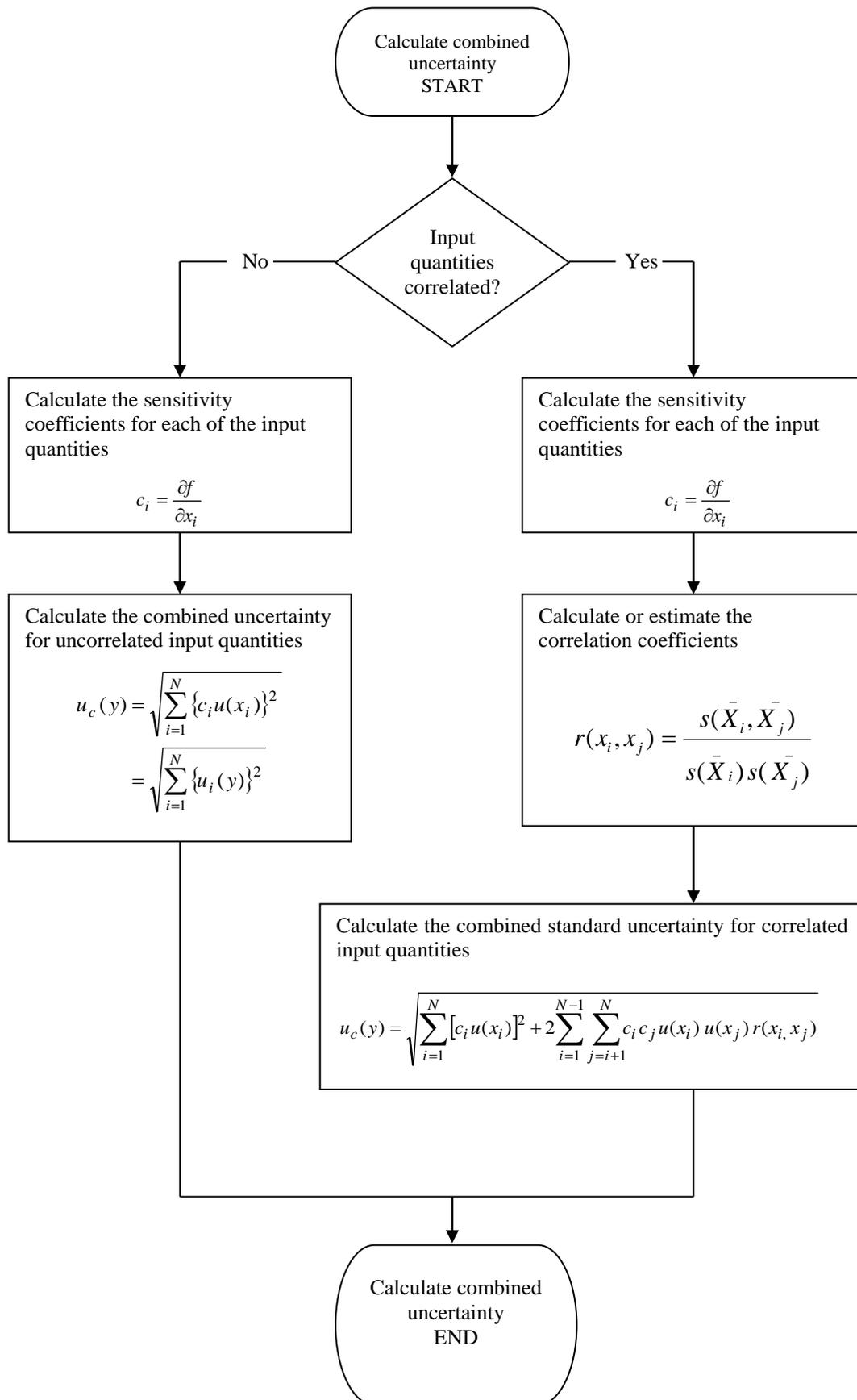
FLOWCHART FOR EVALUATING TYPE A STANDARD UNCERTAINTY



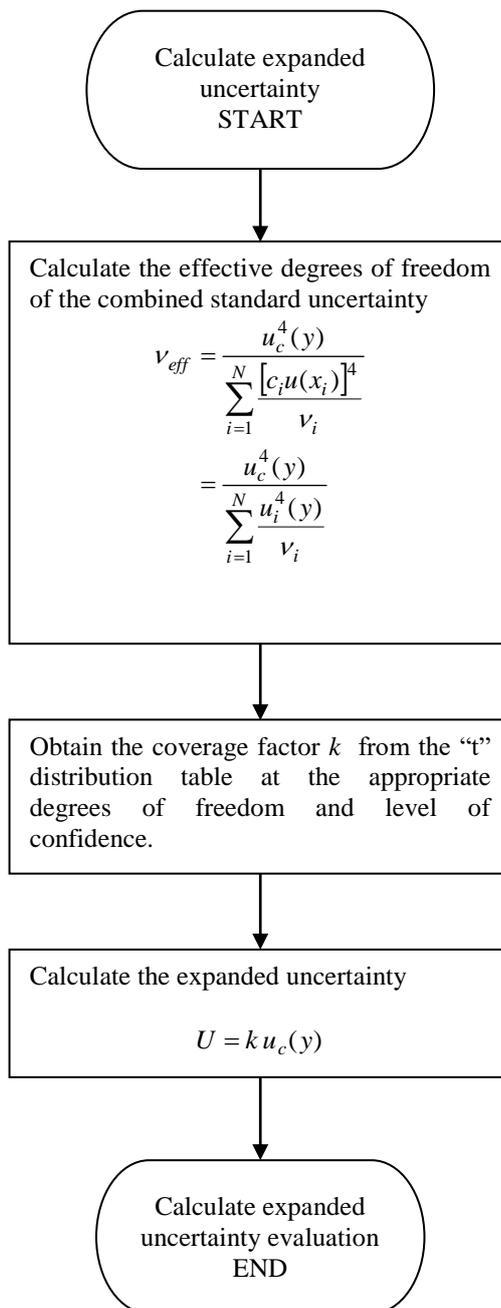
FLOWCHART FOR EVALUATING TYPE B STANDARD UNCERTAINTY



FLOWCHART FOR COMBINING UNCERTAINTY



FLOWCHART FOR CALCULATING EXPANDED UNCERTAINTY



APPENDIX 4
GLOSSARY OF TERMS

a	Half-width of a rectangular distribution of possible values of input quantity X_i : $a = (a_+ - a_-)/2$
a_+	Upper bound, or upper limit, of input quantity X_i
a_-	Lower bound, or lower limit, of input quantity X_i
c_i	Partial derivative or sensitivity coefficient: $c_i \equiv \partial f / \partial x_i$
f	Functional relationship between measurand Y and input quantities X_i on which Y depends, and between output estimate y and input estimates x_i on which y depends
$\partial f / \partial x_i$	Partial derivative with respect to input quantity X_i of functional relationship f between measurand Y and input quantities X_i on which Y depends, evaluated with estimates x_i for the X_i : $\partial f / \partial x_i = \partial f / \partial X_i _{x_1, x_2, \dots, x_N}$
k	Coverage factor used to calculate expanded uncertainty $U = k u_c(y)$ of output estimate y from its combined standard uncertainty $u_c(y)$, where U defines an interval $Y = y \pm U$ having a high level of confidence
n	Number of repeated observations
N	Number of input quantities X_i on which measurand Y depends
P	Probability; level of confidence: $0 \leq p \leq 1$
q	Randomly varying quantity described by a probability distribution
\bar{q}	Arithmetic mean or average of n independent repeated observations q_k of randomly-varying quantity q ; estimate of the expectation or mean μ_q of the probability distribution of q
q_k	k th independent repeated observation of randomly-varying quantity q
$r(x_i, x_j)$	Estimated correlation coefficient associated with input estimates x_i and x_j that estimate input quantities X_i and X_j : $r(x_i, x_j) = u(x_i, x_j) / u(x_i) u(x_j)$
$r(\bar{X}_i, \bar{X}_j)$	Estimated correlation coefficient of input means \bar{X}_i and \bar{X}_j , determined from n independent pairs of repeated simultaneous observations $X_{i,k}$ and $X_{j,k}$ of X_i and X_j ; $r(\bar{X}_i, \bar{X}_j) = s(\bar{X}_i, \bar{X}_j) / s(\bar{X}_i) s(\bar{X}_j)$

**APPENDIX 4
GLOSSARY OF TERMS**

s_p^2	Combined or pooled estimate of variance
s_p	Pooled experimental standard deviation, equal to the positive square root of s_p^2
$s^2(\bar{q})$	Experimental variance of the mean \bar{q} ; estimate of the variance σ^2/n of \bar{q} : $s^2(\bar{q}) = s^2(q_k)/n$; estimated variance obtained from a Type A evaluation
$s(\bar{q})$	Experimental standard deviation of the mean \bar{q} , equal to the positive square root of $s^2(\bar{q})$; $s(\bar{q})$ is a biased estimator of $\sigma(\bar{q})$; standard uncertainty obtained from a Type A evaluation
$s^2(q_k)$	Experimental variance determined from n independent repeated observations q_k of q ; estimate of the variance σ^2 of the probability distribution of q
$s(q_k)$	Experimental standard deviation, equal to the positive square root of $s^2(q_k)$; $s(q_k)$ is a biased estimator of the standard deviation σ of the probability distribution of q
$s(\bar{q}, \bar{r})$	Estimate of the covariance of means \bar{q} and \bar{r} that estimate the expectations μ_q and μ_r of two randomly-varying quantities q and r , determined from n independent pairs of repeated simultaneous observations q_k and r_k of q and r ; estimated covariance obtained from a Type A evaluation
$s(\bar{X}_i, \bar{X}_j)$	Estimate of the covariance of input means \bar{X}_i and \bar{X}_j , determined from n independent pairs of repeated simultaneous observations $X_{i,k}$ and $X_{j,k}$ of X_i and X_j ; estimated covariance obtained from a Type A evaluation
$t_p(\nu)$	t-factor from the t-distribution for ν degrees of freedom corresponding to a given probability p
$t_p(\nu_{eff})$	t-factor from the t-distribution for ν_{eff} degrees of freedom corresponding to a given probability p , used to calculate expanded uncertainty U_p
$u^2(x_i)$	Estimated variance associated with input estimate x_i that estimates input quantity X_i NOTE : When x_i is determined from the arithmetic mean or average of n independent repeated observations, $u^2(x_i) = s^2(\bar{X}_i)$ is an estimated variance obtained from a Type A evaluation

APPENDIX 4
GLOSSARY OF TERMS

$u(x_i)$	Standard uncertainty of input estimate x_i that estimates input quantity X_i , equal to the positive square root of $u^2(x_i)$ NOTE: When x_i is determined from the arithmetic mean or average of n independent repeated observations, $u(x_i) = s(\overline{X}_i)$ is a standard uncertainty obtained from a Type A evaluation
$u(x_i, x_j)$	Estimated covariance associated with two input estimates x_i and x_j that estimate input quantities X_i and X_j NOTE: When x_i and x_j are determined from n independent pairs of repeated simultaneous observations, $u(x_i, x_j) = s(\overline{X}_i, \overline{X}_j)$ is an estimated covariance obtained from a Type A evaluation
$u_c^2(y)$	Combined variance associated with output estimate y
$u_c(y)$	Combined standard uncertainty of output estimate y , equal to the positive square root of $u_c^2(y)$
$u(x_i)/ x_i $	Relative standard uncertainty of input estimate x_i
$[u(x_i)/x_i]^2$	Estimated relative variance associated with input estimate x_i
U	Expanded uncertainty of output estimate y that defines an interval $Y = y \pm U$ having a high level of confidence, equal to coverage factor k times the combined standard uncertainty $u_c(y)$ of y : $U = k u_c(y)$
x_i	Estimate of input quantity X_i NOTE: When x_i is determined from the arithmetic mean or average of n independent repeated observations, $x_i = \overline{X}_i$
X_i	i th input quantity on which measurand Y depends NOTE: X_i may be the physical quantity or the random variable
\overline{X}_i	Estimate of the value of input quantity X_i , equal to the arithmetic mean or average of n independent repeated observations $X_{i,k}$ of X_i
$X_{i,k}$	k th independent repeated observation of X_i
y	Estimate of measurand Y ; result of a measurement; output estimate
Y	A measurand

**APPENDIX 4
GLOSSARY OF TERMS**

$\frac{\Delta u(x_i)}{u(x_i)}$	Estimated relative uncertainty of standard uncertainty $u(x_i)$ of input estimate x_i
μ_q	Expectation or mean of the probability distribution of randomly-varying quantity q
ν	Degrees of freedom (general)
ν_i	Degrees of freedom, or effective degrees of freedom, of standard uncertainty $u(x_i)$ of input estimate x_i
ν_{eff}	Effective degrees of freedom of $u_c(y)$, used to obtain $t_p(\nu_{eff})$ for calculating expanded uncertainty U_p
σ^2	Variance of a probability distribution of (for example) a randomly-varying quantity q , estimated by $s^2(q_k)$
σ	Standard deviation of a probability distribution, equal to the positive square root of σ^2 ; $s(q_k)$ is a biased estimator of σ
$\sigma^2(\bar{q})$	Variance of \bar{q} , equal to σ^2/n , estimated by $s^2(\bar{q}) = s^2(q_k)/n$
$\sigma(\bar{q})$	Standard deviation of \bar{q} , equal to the positive square root of $\sigma^2(\bar{q})$; $s(\bar{q})$ is a biased estimator of $\sigma(\bar{q})$
$\sigma^2[s(\bar{q})]$	Variance of experimental standard deviation $s(\bar{q})$ of \bar{q}
$\sigma[s(\bar{q})]$	Standard deviation of experimental standard deviation $s(\bar{q})$ of \bar{q} , equal to the positive square root of $\sigma^2[s(\bar{q})]$

12 WORKED EXAMPLES

There are a total of fourteen examples in this guide.

EG 1: Digital thermometer calibration

EG 2: Temperature measurement using thermocouple

EG 3: Calibration of platinum resistance thermometer (PRT)

EG 4: DC current measurement

EG 5: Calibration of the DC 20V range of a multimeter

EG 6: Harmonic measurement

EG 7: Power reference

EG 8: Transition time

EG 9: Calibration of an industrial pressure gauge

EG 10: Torque tester calibration

EG 11: Calibration of weighing machine

EG 12: Coordinate measuring machine measurement

EG 13: Micrometer calibration

EG 14: Illumination meter calibration

These examples are used to demonstrate and teach and are not to be considered as definitive analyses for these parameters. The uncertainty contributions in the examples are not defined or recommended or should be since these are dependent on the equipment used and the method of test.

EXAMPLE 1 : DIGITAL THERMOMETER CALIBRATION

1.0 INTRODUCTION

Digital thermometers are widely used in industry to measure the electromotive force (emfs) produced by thermocouples. Modern digital thermometer comes with automatic cold junction compensation (ACJC) with temperature display. A typical schematic circuit diagram used in the calibration of digital thermometer is shown in Figure 1. A reference temperature is set up using the emf source and thermocouple wire. The reference temperature is compared against the temperature indicated in the display of the digital thermometer so as to obtain the correction on the digital thermometer. The calculation of the expanded uncertainty associated with the correction for digital thermometer measuring Type K thermocouple is outlined in this example.

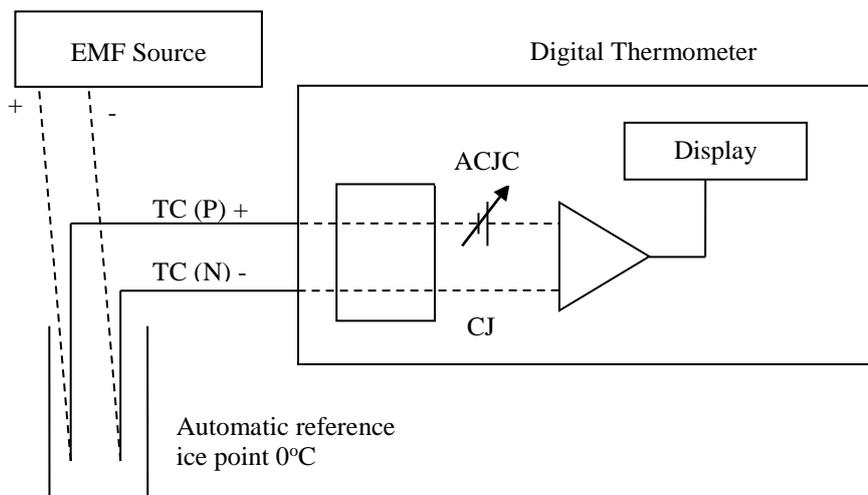


Figure 1: Schematic Diagram for Calibration of Digital Thermometer

where

TC(P)* : Positive leg of the thermocouple

TC(N)* : Negative leg of the thermocouple

CJ : Cold junction of the digital thermometer maintained at ambient temperature.

Note : *Digital thermometer can often be configured to measure different types of thermocouples. The type of thermocouple used in the calibration schematic diagram must be the same as the type configured for measurement in the digital thermometer.

2.0 MATHEMATICAL MODEL

The correction of the digital thermometer is given by the reference temperature minus the displayed temperature of the digital thermometer.

$$C = t_R - t_D \quad \dots(1)$$

where,

C : Correction of the digital thermometer.

t_R : Reference temperature.

t_D : Displayed temperature of the digital thermometer.

EXAMPLE 1 : DIGITAL THERMOMETER CALIBRATION

3.0 UNCERTAINTY EQUATION

The combined standard uncertainty for the correction is given by equation (3). The partial derivatives are equal to one.

$$u_c(C) = \left[\left(\frac{\partial C}{\partial t_R} \right)^2 u^2(t_R) + \left(\frac{\partial C}{\partial t_D} \right)^2 u^2(t_D) \right]^{1/2} \quad \dots(2)$$

$$u_c(C) = [u^2(t_R) + u^2(t_D)]^{1/2} \quad \dots(3)$$

where,

$u_c(C)$: Combined standard uncertainty for correction.

$u(t_R)$: Standard uncertainty for the reference temperature.

$u(t_D)$: Standard uncertainty for the displayed temperature.

4.0 STANDARD UNCERTAINTY EVALUATION

4.1 Determination of the standard uncertainty of the reference temperature $u(t_R)$

The thermocouple wires used in the calibration circuit (Figure 1) have to be calibrated separately since its emf output may deviate from the International Thermocouple Reference Table values. The calibration of the Type K thermocouple wires is reported as having a correction of 0.0097 mV with an expanded uncertainty of ± 0.0015 mV at a confidence level of approximately 95 %, with a coverage factor k equals to 2.

The reference ice point (Figure 1) has to be calibrated separately. The ice point has a zero correction with an expanded uncertainty of ± 0.02 °C at a confidence level of approximately 95 %, with a coverage factor k equals to 2.

The reference emf E_{REF} is given by equation (4).

$$E_{REF} = E_{SOURCE} - \Delta E_{WIRE} - C_0 \Delta t_{ICE} \quad \dots(4)$$

where,

E_{REF} : Reference emf equivalent to a particular ITS-90 temperature.

E_{SOURCE} : Emf output by the emf source.

ΔE_{WIRE} : Emf correction for the calibrated thermocouple wires.

Δt_{ICE} : Temperature correction for the calibrated reference ice-point.

C_0 : Sensitivity coefficient for Type K thermocouple at the reference temperature of 0 °C, 0.0395 mV/ °C.

The standard uncertainty of the reference emf (E_{REF}) is given by equation (5).

$$u(E_{REF}) = [u^2(E_{SOURCE}) + u^2(\Delta E_{WIRE}) + C_0^2 u^2(\Delta t_{ICE})]^{1/2} \quad \dots(5)$$

EXAMPLE 1 : DIGITAL THERMOMETER CALIBRATION

where,

$u(E_{REF})$: Standard uncertainty of the reference emf.

$u(E_{SOURCE})$: Standard uncertainty of the emf source.

$u(\Delta E_{WIRE})$: Standard uncertainty of the correction for the thermocouple wires.

$u(\Delta t_{ICE})$: Standard uncertainty of the correction for the reference ice point.

The emf source used in the calibration has the following one year specifications from the manufacturer.

$$E_{SOURCE} \leq 22 \text{ mV} \quad : \pm 0.0020 \text{ mV}$$

$$22 \text{ mV} < E_{SOURCE} < 50 \text{ mV} \quad : \pm 0.0027 \text{ mV}$$

The uncertainty budget table for the reference emf is given in Table 1.

Table 1

Source of Uncertainty	Symbol X_i	Type	Uncertainty value (mV/°C)	Probability distribution	Coverage factor	Standard uncertainty (u_i) (mV)	Sensitivity coefficient c_i	$ c_i \times u(x_i)$ (mV)	Degrees of freedom ν
Emf Source $E_{SOURCE} \leq 22 \text{ mV}$ $22 \text{ mV} < E_{SOURCE} < 50 \text{ mV}$	E_{SOURCE}	B	0.0020	Rectangular	1.732	0.0012	1	0.0012	∞
		B	0.0027	Rectangular	1.732	0.0016	1	0.0016	∞
TC wires calibration	ΔE_{WIRE}	B	0.0015	Normal	2.000	0.00075	1	0.00075	∞
Reference ice point	Δt_{ICE}	B	0.02	Normal	2.000	0.01	0.0395	0.000395	∞

The standard uncertainty $u(E_{REF})$ of the reference emf is evaluated using equation (5).

for $E_{SOURCE} \leq 22 \text{ mV}$,

$$u(E_{REF}) = \left(0.0012^2 + 0.00075^2 + 0.000395^2 \right)^{1/2}$$

$$= 0.0015 \text{ mV}$$

for $22 \text{ mV} < E_{SOURCE} < 50 \text{ mV}$,

$$u(E_{REF}) = \left(0.0016^2 + 0.00075^2 + 0.000395^2 \right)^{1/2}$$

$$= 0.0019 \text{ mV}$$

The effective degrees of freedom for each of the standard uncertainty is infinity.

The International Thermocouple Reference Table gives the relationship between the temperature and emf. The standard uncertainty of reference temperature $u(t_R)$ can be expressed in terms of the standard uncertainty of the reference emf, $u(E_{REF})$.

$$u(t_R) = \left(\frac{\partial T}{\partial E} \right)_{E_{REF}} u(E_{REF}) \quad \dots(6)$$

EXAMPLE 1 : DIGITAL THERMOMETER CALIBRATION

The $\left(\frac{dT}{dE}\right)_{E_{REF}}$ values are calculated from Type K International Thermocouple Reference Table. For emf equal or less than 22 mV (corresponding to reference temperature from the range -200°C to 500°C), the maximum calculated $\left(\frac{dT}{dE}\right)_{E_{REF}}$ is $\frac{1}{0.015}$ ($^{\circ}\text{C}/\text{mV}$). Whereas, for reference emf greater than 22 mV (corresponding to reference temperature of 600°C to 1200°C), the maximum calculated $\left(\frac{dT}{dE}\right)_{E_{REF}}$ is $\frac{1}{0.036}$ ($^{\circ}\text{C}/\text{mV}$).

The standard uncertainty of reference temperature $u(t_R)$ is given in Table 2. This is evaluated using Equation 6.

Table 2

Source of Uncertainty	$u(E_{REF})$ (mV)	$c_i = \left(\frac{\partial T}{\partial E}\right)_{E_{REF}}$ ($^{\circ}\text{C}/\text{mV}$)	$u(t_R)$ ($^{\circ}\text{C}$)	Degrees of freedom ν_i
Reference emf $E_{REF} \leq 22$ mV (-200°C to 500°C)	0.0015	$\frac{1}{0.015}$	0.100	∞
Reference emf 22 mV $< E_{REF} < 50$ mV (600°C to 1200°C)	0.0019	$\frac{1}{0.036}$	0.053	∞

4.2 Determination of the standard uncertainty in the displayed temperature $u(t_D)$

With a high resolution emf source, it is possible to change the emf signal over a small range of values without producing a change in the displayed reading on the digital thermometer. As a result, there is a range of emf signals to the digital thermometer that gives the same displayed reading. This is one source of uncertainty of a digital instrument due to the resolution of its indicating device.

The maximum value (a_+) and minimum value (a_-) of the emf signals that give the same indication on the digital thermometer are averaged to give the nominal emf value. This can be experimentally performed by slowly approaching each chosen scale mark in one direction and then repeat by slowly approaching from the other direction. The maximum size of this range ($a_+ - a_-$) is usually equals to the resolution of the digital thermometer (δa).

If this technique is used in the calibration, the probability that the nominal emf value lies within the interval a_- to a_+ for all practical purposes is equal to one and the probability that the nominal emf value lies outside this interval is essentially zero. It is assumed that it is equally probable for the nominal emf to lie anywhere within the interval and therefore,

EXAMPLE 1 : DIGITAL THERMOMETER CALIBRATION

a rectangular distribution is assumed with a half width of $\frac{\delta a}{2}$. The standard uncertainty in the displayed reading is given by equation (7).

$$u(t_D) = \left(\frac{\delta a}{2}\right)\left(\frac{1}{\sqrt{3}}\right) \quad \dots(7)$$

For resolution of 0.1 °C and 1.0 °C, $u(t_D)$ is equal to 0.029 °C and 0.29 °C respectively. The degrees of freedom associated with the standard uncertainty of the displayed temperature $u(t_D)$ is infinity.

5.0 CALCULATION OF THE COMBINED STANDARD UNCERTAINTY $u_c(C)$ AND EXPANDED UNCERTAINTY U

The calculations of the combined standard uncertainty and expanded uncertainty associated with the correction for Type K thermocouple are outlined in Table 3. The degrees of freedom of both the standard uncertainties of the reference temperature $u(t_R)$ and the displayed temperature $u(t_D)$ are equal to infinity. As a result, the degrees of freedom for the combined standard uncertainty $u_c(C)$ is also equal to infinity. This gives a coverage factor k equals to 2 at a confidence level of approximately 95 %.

Table 3

Range	$u(t_R)$ (°C)	$u(t_D)$ (°C)	$u_c(C)$ (°C)	$U=k u_c(C)$ (°C)
-200 °C to 500 °C with 0.1 °C resolution 1 °C resolution	0.100	0.029	0.105	0.21
	0.100	0.29	0.307	0.62
600 °C to 1200 °C with 0.1 °C resolution 1 °C resolution	0.053	0.029	0.061	0.13
	0.053	0.29	0.295	0.59

The expanded uncertainty associated with the correction is given by $U = k u_c(C)$ in Table 3 at a confidence level of approximately 95 % with coverage factor k equals to 2.

EXAMPLE 2 : TEMPERATURE MEASUREMENT USING THERMOCOUPLE

1.0 INTRODUCTION

A digital thermometer with a Type K thermocouple are used to measure the temperature inside a temperature chamber. The temperature controller of the chamber is set at 400 °C.

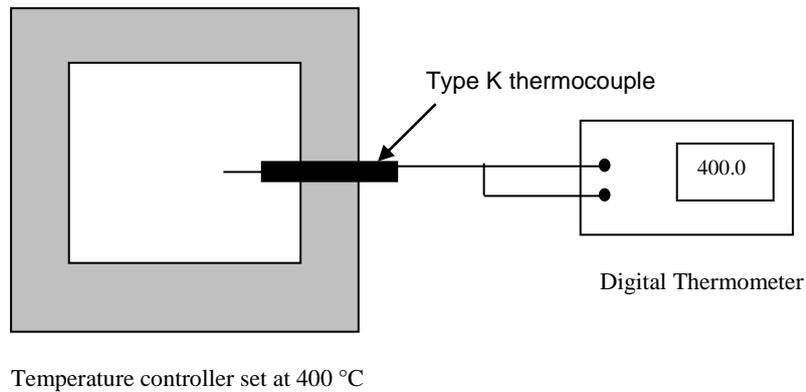


Figure 1 : Measurement set-up

1.1 Digital thermometer specification

Resolution : 0.1 °C

Uncertainty (one year) : ± 0.6 °C

1.2 Thermocouple

The calibration report of the Type K thermocouple gives an uncertainty of ± 1.0 °C at confidence level of approximately 95 % with a coverage factor k equals to 2.0. The correction for the thermocouple at 400 °C is 0.5 °C.

1.3 Measurement record

When the temperature chamber indicator reached 400 °C, the readings are taken after a stabilisation time of half an hour. Ten measurements are taken as given in Table 1.

Table 1

Measurement (i)	T_i (°C)
1	400.1
2	400.0
3	400.1
4	399.9
5	399.9
6	400.0
7	400.1
8	400.2
9	400.0
10	399.9

EXAMPLE 2 : TEMPERATURE MEASUREMENT USING THERMOCOUPLE

2.0 MATHEMATICAL MODEL

The test temperature t_x is given by:

$$t_x \cong t_{rdg} + \Delta t_{tc} + \Delta t_{imm} + \Delta t_{drift} + \Delta t_{ind} + \Delta t_{res} \quad \dots(1)$$

where

t_{rdg} : temperature reading of the Type K thermocouple.

Δt_{tc} : temperature correction of the Type K thermocouple reading based on its calibration data.

Δt_{imm} : temperature correction due to immersion error of the Type K thermocouple.

Δt_{drift} : temperature correction due to drift of the Type K thermocouple.

Δt_{ind} : temperature correction due to deviation of the digital thermometer.

Δt_{res} : temperature correction due to the resolution of the digital thermometer.

3.0 UNCERTAINTY EQUATION

The combined standard uncertainty $u_c(t_x)$ of the test temperature is given by:

$$u_c(t_x) \cong [u^2(t_{rdg}) + u^2(\Delta t_{tc}) + u^2(\Delta t_{imm}) + u^2(\Delta t_{drift}) + u^2(\Delta t_{ind}) + u^2(\Delta t_{res})]^{1/2} \quad \dots(2)$$

where

$u(t_{rdg})$: standard uncertainty of the Type K thermocouple reading.

$u(\Delta t_{tc})$: standard uncertainty of the Type K thermocouple correction.

$u(\Delta t_{imm})$: standard uncertainty of the Type K thermocouple immersion correction.

$u(\Delta t_{drift})$: standard uncertainty of the Type K thermocouple drift correction.

$u(\Delta t_{ind})$: standard uncertainty of the digital thermometer deviation correction.

$u(\Delta t_{res})$: standard uncertainty of the digital thermometer resolution correction.

4.0 STANDARD UNCERTAINTY EVALUATION

4.1 Type A evaluation

4.1.1 Standard uncertainty of the thermocouple reading, $u(t_{rdg})$

$$\text{Mean value, } \bar{T} = \frac{1}{10} \sum_{i=1}^{10} T_i = 400.02 \text{ } ^\circ\text{C}$$

Where T_i are the ten measurements taken as listed in Table 1.

Standard deviation of the mean :

$$s\left(\bar{T}\right) = \frac{s(T_i)}{\sqrt{n}} = \frac{0.103^\circ\text{C}}{\sqrt{10}} = 0.033^\circ\text{C}$$

EXAMPLE 2 : TEMPERATURE MEASUREMENT USING THERMOCOUPLE

Thus the standard uncertainty of the thermocouple reading $u(t_{rdg})$ is $0.033\text{ }^{\circ}\text{C}$.

Degrees of freedom : $\nu = n - 1 = 10 - 1 = 9$

4.2 Type B evaluation

4.2.1 Standard uncertainty of the thermocouple correction $u(\Delta t_{tc})$

The correction for the thermocouple reading is $0.5\text{ }^{\circ}\text{C}$. The standard uncertainty of thermocouple correction $u(\Delta t_{tc})$ is :

$$u(\Delta t_{tc}) = \frac{1.0}{2.0} = 0.500\text{ }^{\circ}\text{C}$$

Degrees of freedom : $\nu = \infty$

4.2.2 Standard uncertainty of the thermocouple immersion correction $u(\Delta t_{imm})$

The uncertainty limit of the thermocouple immersion correction is $\pm 0.1\text{ }^{\circ}\text{C}$. Assuming a rectangular distribution, standard uncertainty of the thermocouple immersion correction $u(\Delta t_{imm})$ is,

$$u(\Delta t_{imm}) = \frac{0.1}{\sqrt{3}} = 0.058\text{ }^{\circ}\text{C}$$

Degrees of freedom : $\nu = \infty$

4.2.3 Standard uncertainty of the thermocouple drift correction $u(\Delta t_{drift})$

The uncertainty limit of the drift is $\pm 0.2\text{ }^{\circ}\text{C}$. Assuming a rectangular distribution, standard uncertainty of the thermocouple drift correction $u(\Delta t_{drift})$ is,

$$u(\Delta t_{drift}) = \frac{0.2}{\sqrt{3}} = 0.115\text{ }^{\circ}\text{C}$$

Degrees of freedom : $\nu = \infty$

4.2.4 Standard uncertainty of the digital thermometer deviation correction $u(\Delta t_{ind})$

From specification, the uncertainty limit of the digital thermometer is $\pm 0.6\text{ }^{\circ}\text{C}$. Assuming a rectangular distribution, the standard uncertainty of the digital thermometer deviation correction $u(\Delta t_{ind})$ is,

$$u(\Delta t_{ind}) = \frac{0.6}{\sqrt{3}} = 0.346\text{ }^{\circ}\text{C}$$

Degrees of freedom : $\nu = \infty$

EXAMPLE 2 : TEMPERATURE MEASUREMENT USING THERMOCOUPLE

4.2.5 Standard uncertainty of the digital thermometer resolution correction $u(\Delta t_{res})$

The half limit due to the resolution of the digital thermometer is 0.05 °C. Assuming a rectangular distribution, the standard uncertainty of the digital thermometer resolution correction $u(\Delta t_{res})$ is,

$$u(\Delta t_{ind}) = \frac{0.05}{\sqrt{3}} = 0.029 \text{ °C}$$

Degrees of freedom : $\nu = \infty$

5.0 UNCERTAINTY BUDGET TABLE

Source of Uncertainty	Symbol (x_i)	Type	Uncertainty Value (°C)	Probability Distribution	Coverage Factor	Standard Uncertainty $u(x_i)$ (°C)	Sensitivity Coefficient c_i	$ c_i \times u(x_i)$ (°C)	Degrees of freedom (ν)
Repeatability of thermocouple reading	$u(t_{rdg})$	A	0.033	-	-	0.033	1	0.033	9
Thermocouple correction	$u(\Delta t_{tc})$	B	1.0	Normal	2.000	0.500	1	0.500	∞
Thermocouple immersion correction	$u(\Delta t_{imm})$	B	0.1	Rectangular	1.732	0.058	1	0.058	∞
Thermocouple drift correction	$u(\Delta t_{arit})$	B	0.2	Rectangular	1.732	0.115	1	0.115	∞
Digital thermometer deviation correction	$u(\Delta t_{ind})$	B	0.6	Rectangular	1.732	0.346	1	0.346	∞
Digital thermometer resolution correction	$u(\Delta t_{res})$	B	0.05	Rectangular	1.732	0.029	1	0.029	∞

6.0 COMBINED STANDARD UNCERTAINTY

The combined standard uncertainty $u_c(t_x)$ of the measured temperature is :

$$\begin{aligned} u_c(t_x) &= \sqrt{0.033^2 + 0.500^2 + 0.058^2 + 0.115^2 + 0.346^2 + 0.029^2} \\ &= 0.623 \text{ °C} \end{aligned}$$

7.0 EFFECTIVE DEGREES OF FREEDOM

Effective degrees of freedom of the combined standard uncertainty is :

$$\begin{aligned} \nu_{eff} &= \frac{0.623^4}{\frac{0.033^4}{9} + \frac{0.500^4}{\infty} + \frac{0.058^4}{\infty} + \frac{0.115^4}{\infty} + \frac{0.346^4}{\infty} + \frac{0.029^4}{\infty}} \\ &\approx \infty \end{aligned}$$

EXAMPLE 2 : TEMPERATURE MEASUREMENT USING THERMOCOUPLE

8.0 EXPANDED UNCERTAINTY

The effective degrees of freedom for the combined standard uncertainty $u_c(t_x)$ was assumed to be infinity. This gives a coverage factor k equals to 2 at a level of confidence of approximately 95 %. Therefore the expanded uncertainty is :

$$\begin{aligned}U &= k u_c(t_x) \\ &= 2 \times 0.623^\circ\text{C} \\ &= 1.3^\circ\text{C}\end{aligned}$$

9.0 REPORTING OF RESULTS

The temperature of the chamber after taking into consideration the correction of the thermocouple is 400.5°C . The measurement uncertainty is $\pm 1.3^\circ\text{C}$, estimated at a level of confidence of approximately 95 % with a coverage factor k equals to 2.

EXAMPLE 3 : CALIBRATION OF PLATINUM RESISTANCE OF THERMOMETER (PRT)

1.0 INTRODUCTION

A semi-standard 100 ohm platinum resistance thermometer (PRT) was calibrated by comparison against a 25 ohm Standard Platinum Resistance Thermometer (SPRT) traceable to the International Temperature Scale 1990 (ITS-90). The calibration was carried out at nominal temperatures of 0 °C, 50 °C, 100 °C, 190 °C, 230 °C, 315 °C and 420 °C using three different types of fluid circulating baths. The schematic diagram of the PRT calibration is as shown in Figure 1.

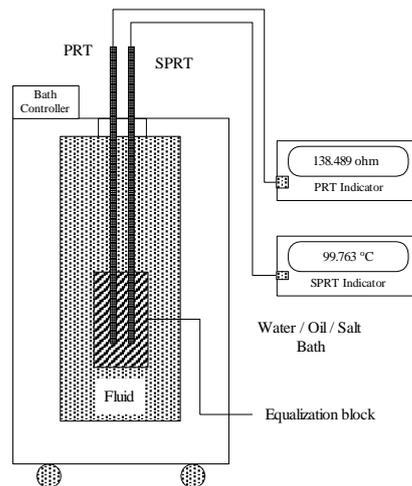


Figure 1: Schematic diagram for PRT calibration

The measurement data are given in Table 1. These data were collected in ascending order of temperatures in degree Celsius. The resistance of the PRT at 0 °C was measured twice, at the beginning and at the end of the calibration using an ice-pot. The temperatures of the baths were measured by the SPRT. The PRT resistance measurements were carried out at 1 mA current that did not include any equivalent resistance due to thermal emf in the PRT.

Table 1

SPRT temperature [$t_{SPRT} : ^\circ\text{C}$]	PRT resistance [$R_{PRT} : \Omega$]
0.000	100.021
50.131	119.498
99.763	138.489
189.869	172.232
229.996	186.955
315.008	217.539
419.573	254.023
0.000	100.019

2.0 MATHEMATICAL MODEL

The uncertainty evaluation was carried out in two parts. Firstly, determination of the test temperature of which the PRT was calibrated and, secondly, determination of the resistance of the PRT at the test temperature.

EXAMPLE 3 : CALIBRATION OF PLATINUM RESISTANCE OF THERMOMETER (PRT)

The test temperature (t_x) was given by:

$$t_x \cong t_{SPRT} + C_{SPRT} \Delta r_{ind} + C_{SPRT} \Delta r_{res} + \Delta t_{rep} + \Delta t_{imm} + \Delta t_{heat} + \Delta t_{drift} + \Delta t_{bath} \dots (1)$$

The resistance (R_x) of the PRT at test temperature was given by:

$$R_x \cong R_{PRT} + \Delta R_{ind} + \Delta R_{res} + \Delta R_{rep} + \Delta R_{imm} + \Delta R_{hys} \dots (2)$$

where

t_{SPRT}	: temperature of the SPRT.
R_{PRT}	: measured resistance of the PRT at test temperature.
$\Delta r_{ind}, \Delta R_{ind}$: resistance correction due to deviation of the indicator of the thermometer.
$\Delta r_{res}, \Delta R_{res}$: resistance correction due to resolution of the indicator of the thermometer.
$\Delta t_{rep}/\Delta R_{rep}$: temperature/resistance correction due to repeatability of the SPRT and PRT.
$\Delta t_{imm}/\Delta R_{imm}$: temperature/resistance correction due to immersion error.
Δt_{heat}	: temperature correction due to self-heating effects of the SPRT.
Δt_{drift}	: temperature correction due to drift of the SPRT since last calibration.
Δt_{bath}	: temperature correction due to non-uniformity of bath temperature.
ΔR_{hys}	: resistance correction due to PRT hysteresis.
C_{SPRT}	: sensitivity coefficient of SPRT ($\partial T/\partial R$).

3.0 UNCERTAINTY EQUATION

The combined standard uncertainty $u_c(t_x)$ of the test temperature was given by:

$$u_c(t_x) \cong \left[\begin{aligned} &u^2(t_{SPRT}) + C_{SPRT}^2 u^2(\Delta r_{ind}) + C_{SPRT}^2 u^2(\Delta r_{res}) + u^2(\Delta t_{rep}) + u^2(\Delta t_{imm}) \\ &+ u^2(\Delta t_{heat}) + u^2(\Delta t_{drift}) + u^2(\Delta t_{bath}) \end{aligned} \right]^{1/2} \dots (3)$$

The combined standard uncertainty $u_c(R_x)$ of the PRT resistance at test temperature was given by:

$$u_c(R_x) \cong \left[\frac{u_c^2(t_x)}{C_{PRT}^2} + u^2(\Delta R_{ind}) + u^2(\Delta R_{res}) + u^2(\Delta R_{rep}) + u^2(\Delta R_{imm}) + u^2(\Delta R_{hys}) \right]^{1/2} \dots (4)$$

where

$u(t_{SPRT})$: standard uncertainty of the SPRT.
$u(R_{PRT})$: standard uncertainty of measured resistance of the PRT at test temperature was given by the combined standard uncertainty ($u_c(t_x)$) of the test temperature evaluated in Equation 3 divided by the sensitivity coefficient of PRT (C_{PRT}). ($= u_c(t_x)/C_{PRT}$).
$u(\Delta r_{ind}), u(\Delta R_{ind})$: standard uncertainty of the indicator correction.
$u(\Delta r_{res}), u(\Delta R_{res})$: standard uncertainty of the indicator's resolution correction.

EXAMPLE 3 : CALIBRATION OF PLATINUM RESISTANCE OF THERMOMETER (PRT)

- $u(\Delta t_{rep}), u(\Delta R_{rep})$: standard uncertainty of the SPRT and PRT repeatability.
 $u(\Delta t_{imm}), u(\Delta R_{imm})$: standard uncertainty of the SPRT and PRT immersion correction.
 $u(\Delta t_{heat})$: standard uncertainty of the SPRT self-heating effects correction.
 $u(\Delta t_{drift})$: standard uncertainty of the SPRT drift correction.
 $u(\Delta t_{bath})$: standard uncertainty of the temperature correction for bath non-uniformity bath.
 $u(\Delta R_{hys})$: standard uncertainty of the PRT hysteresis correction.
 c_{PRT} : sensitivity coefficient of PRT ($\partial T / \partial R$).

The typical sensitivity coefficient (c_{SPRT}) of a 25 ohm SPRT is 10.0 °C/Ω, whereas the typical sensitivity coefficient (c_{PRT}) of a 100 Ω PRT is 2.5 °C/Ω, and the inverse of c_{PRT} ($1/c_{PRT}$), as required in Equation 4 is 0.4 Ω/°C.

4.0 EVALUATION OF UNCERTAINTY

No correction was applied to any of the correction terms stated in Equation 1 and 2.

Symbol	Information	Uncertainty value
t_{SPRT}	The SPRT calibration certificate stated that expanded uncertainty (U) was ± 0.004 °C with a coverage factor $k = 2$.	± 0.004 °C
$\Delta r_{ind} / \Delta R_{ind}$	The specifications of the SPRT and PRT indicators were ± 0.0002 Ω and ± 0.008 Ω respectively.	± 0.0002 Ω ^[1] / ± 0.008 Ω
$\Delta r_{res} / \Delta R_{res}$	No correction was applied due to the resolution of the indicator. The uncertainty value associated with this correction was taken to be half of the least significant digit of the indicator. SPRT indicator's resolution: 0.0001 Ω, PRT indicator's resolution : 0.001 Ω	± 0.00005 Ω / ± 0.0005 Ω
$\Delta t_{rep} / \Delta R_{rep}$	The maximum standard errors of the average readings of the SPRT and PRT were ± 0.002 °C (± 0.0002 Ω) and ± 0.001 Ω respectively.	± 0.002 °C ^[2] / ± 0.001 Ω
$\Delta t_{imm} / \Delta R_{imm}$	The immersion depth during the calibration was 300 mm. Both the SPRT and PRT were sufficiently immersed into the circulating baths such that the heat leakage from or to the sensing element was negligible. An uncertainty limit of ± 0.001 °C was estimated for the SPRT and ± 0.001 Ω for the PRT.	± 0.001 °C / ± 0.001 Ω
Δt_{heat}	The SPRT was calibrated at 1 mA measuring current. No correction was applied to the SPRT due to the self-heating effects contributed by the 1 mA measuring current at the time of usage. An uncertainty limit of ± 0.001 °C was estimated.	± 0.001 °C

EXAMPLE 3 : CALIBRATION OF PLATINUM RESISTANCE OF THERMOMETER (PRT)

Symbol	Information	Uncertainty value
Δt_{drift}	No correction was made for the SPRT drift since the last calibration. The uncertainty limit of this correction was ± 0.002 °C based on the previous calibrations.	± 0.002 °C
Δt_{bath}	No correction was applied for the bath non uniformity inside the calibration baths. The uncertainty limit of this correction value was found to be within ± 0.005 °C.	± 0.005 °C
ΔR_{hys}	No correction was applied for the hysteresis effect. The uncertainty of the hysteresis correction was estimated to be the change in the PRT resistance at 0.0 °C, which was 0.002 Ω .	± 0.002 Ω

Remarks

[1] The calibration certificates of the indicators results showed that the errors of the indicators and calibration uncertainties were within the specifications of the indicators. No correction was applied to the indicator resistance readings. Drift and other influences such as environmental effects were considered negligible, therefore only the specifications of the indicators were included.

[2] The SPRT and PRT readings were taken simultaneously in order to minimise the error caused by the temporal fluctuations of the bath. After the bath had stabilised, thirty SPRT and PRT readings at a nine seconds interval were observed.

5.0 UNCERTAINTY BUDGET TABLE

Uncertainty budget table for the test temperature (t_x) was tabulated below.

Source of Uncertainty	Symbol x_i	Type	Uncertainty Value	Probability Distribution	Coverage Factor	Standard uncertainty $u(x_i)$	Sensitivity Coefficient c_i	$ c_i x u(x_i)$ (°C)	Degrees of freedom ν
SPRT	t_{SPRT}	B	0.004 °C	Normal	2	0.0020	1	0.0020	∞
SPRT indicator specifications	Δr_{ind}	B	0.0002 Ω	Rectangular	1.732	0.00012	10	0.0012	∞
SPRT indicator resolution	Δr_{res}	B	0.00005 Ω	Rectangular	1.732	0.000029	10	0.00029	∞
SPRT repeatability	Δt_{rep}	A	0.002 °C	-	-	0.0020	1	0.0020	29
SPRT immersion	Δt_{imm}	B	0.001 °C	Rectangular	1.732	0.00058	1	0.00058	∞
SPRT self-heating	Δt_{heat}	B	0.001 °C	Rectangular	1.732	0.00058	1	0.00058	∞
SPRT drift	Δt_{drift}	B	0.002 °C	Rectangular	1.732	0.0012	1	0.0012	∞
Bath non-uniformity	Δt_{bath}	B	0.005 °C	Rectangular	1.732	0.0029	1	0.0029	∞

EXAMPLE 3 : CALIBRATION OF PLATINUM RESISTANCE OF THERMOMETER (PRT)

Uncertainty budget table for the PRT resistance (R_x) at test temperature (t_x) was tabulated below.

Source of Uncertainty	Symbol x_i	Type	Uncertainty Value	Probability Distribution	Coverage factor	Standard uncertainty $u(x_i)$	Sensitivity Coefficient c_i	$ c_i x u(x_i)$ (Ω)	Degrees of freedom ν
Test temperature	t_x	B	0.0045 °C	Normal	1	0.0045	0.4	0.0018	700
PRT indicator specifications	ΔR_{ind}	B	0.008 Ω	Rectangular	1.732	0.0046	1	0.0046	∞
PRT indicator resolution	ΔR_{res}	B	0.0005 Ω	Rectangular	1.732	0.00029	1	0.00029	∞
PRT repeatability	ΔR_{rep}	A	0.001 Ω	-	-	0.0010	1	0.0010	29
PRT immersion	ΔR_{imm}	B	0.001 Ω	Rectangular	1.732	0.00058	1	0.00058	∞
PRT hysteresis	ΔR_{hys}	B	0.002 Ω	Rectangular	1.732	0.0012	1	0.0012	∞

6.0 COMBINED STANDARD UNCERTAINTY

The combined standard uncertainty $u_c(t_x)$ of the test temperature (t_x) was given below.

$$u_c(t_x) = \left[0.0020^2 + 0.0012^2 + 0.00029^2 + 0.0020^2 + 0.00058^2 + 0.00058^2 + 0.0012^2 + 0.0029^2 \right]^{1/2} = 0.0045 \text{ °C}$$

The combined standard uncertainty $u_c(R_x)$ of the PRT resistance was given below.

$$u_c(R_x) = \left[0.0018^2 + 0.0046^2 + 0.00029^2 + 0.0010^2 + 0.00058^2 + 0.0012^2 \right]^{1/2} = 0.0053 \text{ } \Omega$$

7.0 EFFECTIVE DEGREES OF FREEDOM

Effective degrees of freedom of the combined standard uncertainty $u_c(t_x)$ was :

$$\nu_{eff} = \frac{0.0045^4}{\frac{0.0020^4}{\infty} + \frac{0.0012^4}{\infty} + \frac{0.00029^4}{\infty} + \frac{0.0020^4}{29} + \frac{0.00058^4}{\infty} + \frac{0.00058^4}{\infty} + \frac{0.0012^4}{\infty} + \frac{0.0029^4}{\infty}} \approx 700$$

Effective degrees of freedom of the combined standard uncertainty $u_c(R_x)$ was :

$$\nu_{eff} = \frac{0.0053^4}{\frac{0.0018^4}{700} + \frac{0.0046^4}{\infty} + \frac{0.00029^4}{\infty} + \frac{0.0010^4}{29} + \frac{0.00058^4}{\infty} + \frac{0.0012^4}{\infty}} \approx 15000$$

EXAMPLE 3 : CALIBRATION OF PLATINUM RESISTANCE OF THERMOMETER (PRT)

8.0 EXPANDED UNCERTAINTY

The effective degrees of freedom for the combined standard uncertainty $u_c(R_x)$ was taken to be infinity. At a level of confidence of approximately 95 %, a coverage factor k of 2 was used. The expanded uncertainty,

$$\begin{aligned} U &= k u_c(R_x) \\ &= 2 \times 0.0053 \Omega \\ &= 0.011 \Omega \end{aligned}$$

9.0 REPORTING OF RESULTS

Test temperature [t_x : °C]	PRT resistance [R_x : Ω]
0.000	100.020
50.131	119.498
99.763	138.489
189.869	172.232
229.996	186.955
315.008	217.539
419.573	254.023

The expanded uncertainty of the PRT resistance given at the test temperatures was $\pm 0.011 \Omega$, estimated at a level of confidence of approximately 95 % with a coverage factor of $k = 2$.

Remark :

The following cubic equation was used to interpolate the PRT resistance from 0 °C to 420 °C, where $W(t) = R(t) / R(0 \text{ °C})$. The coefficients a , b and c were determined by fitting the calibration results using the method of least squares.

$$W(t) = 1 + a \left(\frac{t}{100} \right) + b \left(\frac{t}{100} \right)^2 + c \left(\frac{t}{100} \right)^3$$

$$a = 3.914069 \times 10^{-1}, b = -5.918921 \times 10^{-3}, c = 2.281030 \times 10^{-5}$$

EXAMPLE 5 : CALIBRATION OF DC 20 V RANGE OF A MULTIMETER

1.0 INTRODUCTION

A current of 10A is measured by using a current shunt and a voltmeter. The specifications of the instruments used are as follows:

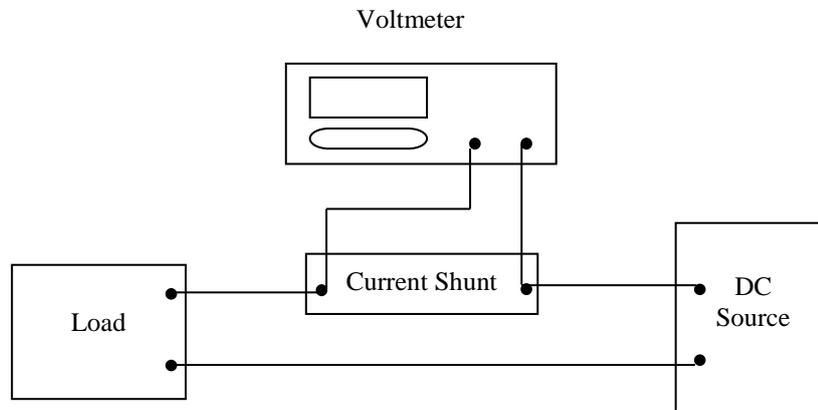


Figure 1: Measurement set-up

1.1 Current Shunt Specifications

- Current : 10 A
- Resistance : 0.01 Ω
- The current shunt was calibrated recently. From the calibration report, we have the following:

Resistance $R=0.010088 \Omega$ measured at 10 A, 23 °C

Relative expanded uncertainty : $\pm 0.08 \%$ at coverage factor $k = 2$

Temperature coefficient in the range 15 °C to 30 °C: 60 ppm / K

The relative expanded uncertainties can be converted into expanded uncertainty by multiplying by R.

- The uncertainty due to drift of resistance value was found negligible.

1.2 Voltmeter Specifications

- All uncertainty¹ specifications apply for one year after calibration when operated in a temperature of 15 °C to 40 °C and a relative humidity of up to 80 %.
- DC voltage function specification

Range	Full scale	Uncertainty ¹ ± (% of reading + number of counts)
200 mV	199.99 mV	0.030 + 2

Note : ¹ Traditionally the term accuracy is used

EXAMPLE 5 : CALIBRATION OF DC 20 V RANGE OF A MULTIMETER

1.3 Measurement Record

Room temperature : $(23 \pm 5) ^\circ\text{C}$, Humidity : $(55 \pm 10) \%$ R.H.

Reading no.	Voltage (mV)
1	100.68
2	100.83
3	100.79
4	100.64
5	100.63
6	100.94
7	100.60
8	100.68
9	100.76
10	100.65

2.0 MATHEMATICAL MODEL

The current is a function of the voltage and resistance. From Ohm's law,

$$\begin{aligned} I &= f(x_1, x_2) \\ &= f(V, R) \\ &= \frac{V}{R} \end{aligned} \quad \dots(1)$$

where I is the current, V is the voltage, and R is the resistance.

The best estimate of the voltage V is the mean voltage \bar{V} of a series of voltage measurement.

3.0 UNCERTAINTY EQUATION

For uncorrelated input quantities, the combined standard uncertainty $u_c(I)$ of the current measured is :

$$u_c^2(I) = \sum_{i=1}^N \left(\frac{\partial f}{\partial x_i} \right)^2 u^2(x_i) \quad \dots(2)$$

EXAMPLE 5 : CALIBRATION OF DC 20 V RANGE OF A MULTIMETER

Major sources of uncertainties in the measurement include:

- (1) Uncertainties due to the measured voltage which consists of the random variability of the measured voltage and the voltmeter uncertainty. The effect of the room temperature on voltmeter has already been taken into account in the error limits.
- (2) Uncertainties due to the current shunt resistance which consist of the calibrated resistance value and the resistance change due to temperature effect. The uncertainty due to drift of resistance value is negligible.

Let ,

$u_1(V)$: Standard uncertainty of voltage due to random effect.

$u_2(V)$: Standard uncertainty of voltage due to voltmeter uncertainty.

$u_3(R)$: Standard uncertainty of current shunt calibrated resistance value.

$u_4(R)$: Standard uncertainty of current shunt resistance due to temperature effect.

Then,

$$\begin{aligned}u_c^2(I) &= \left(\frac{\partial I}{\partial V}\right)^2 u_1^2(V) + \left(\frac{\partial I}{\partial V}\right)^2 u_2^2(V) + \left(\frac{\partial I}{\partial R}\right)^2 u_3^2(R) + \left(\frac{\partial I}{\partial R}\right)^2 u_4^2(R) \\&= [c_1 \cdot u_1(V)]^2 + [c_1 \cdot u_2(V)]^2 + [c_2 \cdot u_3(R)]^2 + [c_2 \cdot u_4(R)]^2 \\&= (c_1)^2 [u_1^2(V) + u_2^2(V)] + (c_2)^2 [u_3^2(R) + u_4^2(R)] \quad \dots(3)\end{aligned}$$

and the sensitivity coefficients are :

$$c_1 \equiv \frac{\partial I}{\partial V} = \frac{1}{R} \quad \dots(4)$$

$$c_2 \equiv \frac{\partial I}{\partial R} = -\frac{V}{R^2} \quad \dots(5)$$

4.0 ANALYSIS OF UNCERTAINTY COMPONENTS

4.1 Type A Evaluation

4.1.1 Voltage

The data are tabulated in Table 1 for the calculation of mean voltage and standard deviation of the voltage.

EXAMPLE 5 : CALIBRATION OF DC 20 V RANGE OF A MULTIMETER

Table 1: Data for calculation of mean and standard deviation of voltage

No.	V_i (mV)	$(V_i - \bar{V}) \times 10^{-2}$	$(V_i - \bar{V})^2 \times 10^{-4}$
1	100.68	-4	16
2	100.83	11	121
3	100.79	7	49
4	100.64	-8	64
5	100.63	-9	81
6	100.94	22	484
7	100.60	-12	144
8	100.68	-4	16
9	100.76	4	16
10	100.65	-7	49
Total	1007.2	0	1040

Mean voltage:

$$\begin{aligned} \bar{V} &= \frac{\sum_{i=1}^n V_i}{n} \\ &= \frac{1007.20}{10} \\ &= 100.72 \text{ mV} \end{aligned} \quad \dots(6)$$

The best estimate of the voltage is therefore :

$$V = \bar{V} = 100.72 \text{ mV} \quad \dots(7)$$

Standard deviation:

$$\begin{aligned} s(V_i) &= \sqrt{\left(\frac{1}{n-1}\right) \sum_{i=1}^n (V_i - \bar{V})^2} \\ &= \sqrt{\left(\frac{1}{9}\right) (1040 \times 10^{-4})} \\ &= 10.75 \times 10^{-2} \text{ mV} \end{aligned} \quad \dots(8)$$

EXAMPLE 5 : CALIBRATION OF DC 20 V RANGE OF A MULTIMETER

Standard deviation of the mean:

$$\begin{aligned} s(\bar{V}) &= \frac{s(V_i)}{\sqrt{n}} \\ &= \frac{10.75 \times 10^{-2}}{\sqrt{10}} \\ &= 3.40 \times 10^{-2} \text{ mV} \end{aligned} \quad \dots(9)$$

Standard uncertainty of voltage due to random effect is:

$$u_1(V) = 3.40 \times 10^{-2} \text{ mV} \quad \dots(10)$$

Degrees of freedom :

$$\begin{aligned} \nu_1 &= n - 1 \\ &= 10 - 1 \\ &= 9 \end{aligned} \quad \dots(11)$$

4.2 Type B Evaluation

4.2.1 Uncertainty of Voltmeter

From specification, the uncertainty of the voltmeter is:

$$\begin{aligned} a_2 &= (0.030 \% \text{ of reading} + 2 \text{ counts}) \\ &= (0.030 \% \times 100.72 \text{ mV} + 0.02 \text{ mV}) \\ &= 5.02 \times 10^{-2} \text{ mV} \end{aligned} \quad \dots(12)$$

Assumed rectangular distribution, the standard uncertainty of voltage due to voltmeter uncertainty is:

$$\begin{aligned} u_2(V) &= \frac{a_2}{\sqrt{3}} \\ &= \frac{5.02 \times 10^{-2}}{\sqrt{3}} \\ &= 2.90 \times 10^{-2} \text{ mV} \end{aligned} \quad \dots(13)$$

Degrees of freedom:

$$\nu_2 = \infty \quad \dots(14)$$

EXAMPLE 5 : CALIBRATION OF DC 20 V RANGE OF A MULTIMETER

4.2.2 Current Shunt Resistance

From calibration report, the uncertainty of the shunt resistance is:

$$\begin{aligned} a_3 &= 8 \times 10^{-4} \times R \\ &= 8 \times 10^{-4} \times 0.010088 \\ &= 8.07 \times 10^{-6} \Omega \end{aligned} \quad \dots(15)$$

Given coverage factor $k = 2$, the standard uncertainty of current shunt resistance is:

$$\begin{aligned} u_3(R) &= \frac{a_3}{k} \\ &= \frac{8.07 \times 10^{-6}}{2} \\ &= 4.04 \times 10^{-6} \Omega \end{aligned} \quad \dots(16)$$

Degrees of freedom:

$$\nu_3 = \infty \quad \dots(17)$$

4.2.3 Temperature Effect on Current Shunt ($23 \pm 5 \text{ }^\circ\text{C}$)

From the calibration report, the uncertainty of the shunt resistance due to the effect of room temperature $\Delta t = 5 \text{ }^\circ\text{C}$ is:

$$\begin{aligned} a_4 &= 60 \times 10^{-6} \times \Delta t \times R \\ &= 60 \times 10^{-6} \times 5 \times 0.010088 \\ &= 3.03 \times 10^{-6} \Omega \end{aligned} \quad \dots(18)$$

Assumed rectangular distribution, the standard uncertainty of resistance due to the temperature effect is:

$$\begin{aligned} u_4(R) &= \frac{a_4}{\sqrt{3}} \\ &= \frac{3.03 \times 10^{-6}}{\sqrt{3}} \\ &= 1.75 \times 10^{-6} \Omega \end{aligned} \quad \dots(19)$$

Degrees of freedom:

$$\nu_4 = \infty \quad \dots(20)$$

EXAMPLE 5 : CALIBRATION OF DC 20 V RANGE OF A MULTIMETER

5.0 CALCULATION OF CURRENT

$$\begin{aligned}
 I &= \frac{V}{R} \\
 &= \frac{100.72 \times 10^{-3}}{0.010088} \\
 &= 9.984 \text{ A} \qquad \dots(21)
 \end{aligned}$$

6.0 SENSITIVITY COEFFICIENTS

$$\begin{aligned}
 c_1 &= \frac{1}{R} \\
 &= \frac{1}{0.010088} \\
 &= 99.128 \Omega^{-1} \qquad \dots(22)
 \end{aligned}$$

$$\begin{aligned}
 c_2 &= -\frac{V}{R^2} \\
 &= -\frac{100.72 \times 10^{-3}}{0.001088} \\
 &= -989.70 \text{ V}/\Omega^2 \qquad \dots(23)
 \end{aligned}$$

7.0 UNCERTAINTY BUDGET

Source of Uncertainty	Symbol x_i	Type	Uncertainty Value	Probability Distribution	Coverage Factor	Standard Uncertainty $u(x_i)$	Sensitivity Coefficient c_i	$ c_i \times u(x_i)$ (A)	Degrees of Freedom ν
Repeatability	$u_1(V)$	A	3.40×10^{-2} mV	-	-	3.40×10^{-2} mV	$99.128 \Omega^{-1}$	3.37×10^{-3}	9
Voltmeter	$u_2(V)$	B	5.02×10^{-2} mV	Rectangular	$\sqrt{3}$	2.90×10^{-2} mV	$99.128 \Omega^{-1}$	2.87×10^{-3}	∞
Shunt Resistance	$u_3(R)$	B	8.07×10^{-6} Ω	Normal	2	4.04×10^{-6} Ω	$989.7 \text{ V}/\Omega^2$	3.99×10^{-3}	∞
Shunt Temperature Effect	$u_4(R)$	B	3.03×10^{-6} Ω	Rectangular	$\sqrt{3}$	1.75×10^{-6} Ω	$989.7 \text{ V}/\Omega^2$	1.73×10^{-3}	∞

8.0 COMBINED STANDARD UNCERTAINTY $u_c(y)$

For uncorrelated input quantities:

$$u_c^2(I) = c_1^2 [u_1^2(V) + u_2^2(V)] + c_2^2 [u_3^2(R) + u_4^2(R)] \qquad \dots(24)$$

EXAMPLE 5 : CALIBRATION OF DC 20 V RANGE OF A MULTIMETER

Then,

$$u_c^2(I) = (99.128)^2 \left[(3.40 \times 10^{-2})^2 + (2.90 \times 10^{-2})^2 \right] (10^{-3})^2 \\ + (-989.70)^2 \left[(4.04 \times 10^{-6})^2 + (1.75 \times 10^{-6})^2 \right] \quad \dots(25)$$

Hence,

$$u_c(I) = \sqrt{3.86 \times 10^{-5}} \\ = 6.2 \times 10^{-3} \text{ A} \quad \dots(26)$$

9.0 EFFECTIVE DEGREES OF FREEDOM ν_{eff}

$$\nu_{eff} = \frac{u_c^4(I)}{\frac{[c_1 u_1(V)]^4}{\nu_1} + \frac{[c_1 u_2(V)]^4}{\nu_2} + \frac{[c_1 u_3(V)]^4}{\nu_3} + \frac{[c_1 u_4(V)]^4}{\nu_4}} \\ = \frac{(6.2 \times 10^{-3})^4}{\frac{(99.128 \times 3.40 \times 10^{-5})^4}{9} + \frac{(99.128 \times 2.90 \times 10^{-5})^4}{\infty} + \frac{(-989.70 \times 4.04 \times 10^{-6})^4}{\infty} + \frac{(-989.70 \times 1.75 \times 10^{-6})^4}{\infty}} \\ \approx 103 \quad \dots(27)$$

10.0 EXPANDED UNCERTAINTY U

For effective degrees of freedom $\nu_{eff} > 100$, $k = 2$ can be used at approximately 95 % level of confidence. Therefore, the expanded uncertainty U is :

$$U = k u_c(I) \\ = 2 \times 6.2 \times 10^{-2} \\ = 0.012 \text{ A} \quad \dots(28)$$

11.0 REPORTING OF RESULTS

$$I = (9.984 \pm 0.012) \text{ A} \quad \dots(29)$$

The measured result is 9.984 A. The reported expanded uncertainty is ± 0.012 A with a coverage factor of $k = 2$, assuming a normal distribution at a level of confidence of approximately 95 %.

EXAMPLE 5 : CALIBRATION OF DC 20 V RANGE OF A MULTIMETER

12.0 REMARKS

12.1 Alternative method to calculate combined standard uncertainty u_c by relative combined method
 Since the mathematical model is of the form

$$Y = c X_1^{P_1} X_2^{P_2} \dots X_N^{P_N} \quad \dots(30)$$

The combined standard uncertainty $u_c(y)$ can be expressed as an estimated relative combined variance:

$$\left(\frac{u_c(y)}{y}\right)^2 = \sum_{i=1}^N \left(\frac{u(x_i)}{x_i}\right)^2 \quad \dots(31)$$

Therefore,

$$\begin{aligned} \left(\frac{u_c(I)}{I}\right)^2 &= \left(\frac{u_1(V)}{V}\right)^2 + \left(\frac{u_2(V)}{V}\right)^2 + \left(\frac{u_3(R)}{R}\right)^2 + \left(\frac{u_4(R)}{R}\right)^2 \\ &= \left(\frac{3.40 \times 10^{-2}}{100.72}\right)^2 + \left(\frac{2.90 \times 10^{-2}}{100.72}\right)^2 + \left(\frac{4.04 \times 10^{-6}}{0.010088}\right)^2 + \left(\frac{1.75 \times 10^{-6}}{0.010088}\right)^2 \\ &= 3.87 \times 10^{-7} \text{ per unit}^2 \end{aligned} \quad \dots(32)$$

Then,

$$\left(\frac{u_c(I)}{I}\right) = 6.2 \times 10^{-4} \text{ (per unit)} \quad \dots(33)$$

And

$$\begin{aligned} u_c(I) &= I \times 6.2 \times 10^{-4} \\ &= 9.984 \times 6.2 \times 10^{-4} \\ &= 6.2 \times 10^{-3} \text{ A} \end{aligned} \quad \dots(34)$$

12.2 Alternative method to calculate effective degrees of freedom,

$$\begin{aligned} \nu_{eff} &= \frac{\left[\frac{u_c(I)}{I}\right]^4}{\frac{\left[\frac{u_1(V)}{V}\right]^4}{\nu_1} + \frac{\left[\frac{u_2(V)}{V}\right]^4}{\nu_2} + \frac{\left[\frac{u_3(R)}{R}\right]^4}{\nu_3} + \frac{\left[\frac{u_4(R)}{R}\right]^4}{\nu_4}} \\ &= \frac{\left[\frac{6.2 \times 10^{-3}}{9.984}\right]^4}{\frac{\left[\frac{3.40 \times 10^{-2}}{100.72}\right]^4}{9} + \frac{\left[\frac{2.90 \times 10^{-2}}{100.72}\right]^4}{\infty} + \frac{\left[\frac{4.04 \times 10^{-6}}{0.010088}\right]^4}{\infty} + \frac{\left[\frac{1.75 \times 10^{-6}}{0.010088}\right]^4}{\infty}} \\ &\approx 103 \end{aligned} \quad \dots(35)$$

EXAMPLE 5 : CALIBRATION OF DC 20 V RANGE OF A MULTIMETER

1.0 INTRODUCTION

The manufacturer's calibration procedure of a multi-meter required that a DC voltage of 10 V from a multifunction calibrator is applied to the multi-meter to verify its DC 20 V range accuracy. The specifications of the multifunction calibrator and digital multi-meter are as follows:

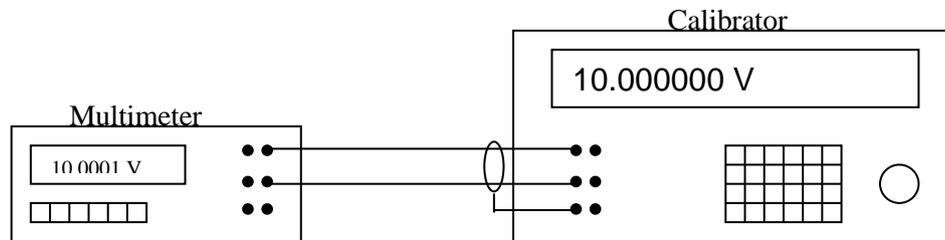


Figure 1 : Connection diagram

1.1 Multifunction Calibrator Specifications

The calibrator is calibrated every 90 days. The last calibration was carried out at a 23 °C ± 1 °C environment and the calibrator was verified to be within specifications.

DC Voltage Specifications

Range	Resolution	Uncertainty at 99 % level of confidence ± 5 °C from calibration temperature		
		24 Hours	90 Days	1 Year
		± (ppm output + μV)		
20 V	1 μV	4 + 3	5 + 4	8 + 5

1.2 Digital Multimeter Specifications

DC Voltage range

Range	Full Scale Display	Resolution	1 Year Uncertainty ¹ ± (% of reading + % of range)
20 V	19.9999 V	100 μV	0.0035 + 0.0025

Note : ¹ Traditionally the term accuracy is used

It was given in the service manual that a DC voltage of 10 V is used to verify the multimeter's DC 20 V range. The verification specification for the multimeter DC 20 V range is:

EXAMPLE 5 : CALIBRATION OF DC 20 V RANGE OF A MULTIMETER

Verification specification

Range	Input Voltage	Limits
20 V	10 V	± 0.0007 V

1.3 Measurement Record

Temperature : $23\text{ }^{\circ}\text{C} \pm 2\text{ }^{\circ}\text{C}$

Only one reading was taken as the indicated voltage remain unchanged. Errors due to loading effect and connection are negligible.

Applied Voltage	Indicated Voltage
10.000000 V	10.0001 V

2.0 MATHEMATICAL MODEL

The multimeter was connected directly to the calibrator. The model of the process is a function of voltages :

$$\begin{aligned} V_{DMM} &= f(V) \\ &= V_{STD} + \Delta V_{DMM} \end{aligned} \quad \dots(1)$$

where,

V_{DMM} : Multimeter indicated voltage.

V_{STD} : Applied voltage from the calibrator.

ΔV_{DMM} : Error of the multimeter.

3.0 UNCERTAINTY EQUATION

The input quantities are uncorrelated. The combined standard uncertainty $u_c(V)$ is :

$$u_c^2(V) = \sum_{i=1}^N \left(\frac{\partial f}{\partial V_i} \right)^2 u^2(V) \quad \dots(2)$$

The voltage from the calibrator is directly applied to the multimeter. Since errors due to loading effect and connection were found negligible, the components of the total measurement uncertainty consist of :

$u_1(V)$: Calibrator's applied voltage uncertainty.

$u_2(V)$: Multimeter's random effect uncertainty.

EXAMPLE 5 : CALIBRATION OF DC 20 V RANGE OF A MULTIMETER

The combined standard uncertainty becomes :

$$\begin{aligned}u_c^2(V) &= \left(\frac{\partial V_{DMM}}{\partial V_{STD}} \right)^2 u_1^2(V) + \left(\frac{\partial V_{DMM}}{\partial \Delta V_{DMM}} \right)^2 u_2^2(V) \\ &= (c_1)^2 u_1^2(V) + (c_2)^2 u_2^2(V) \quad \dots(3)\end{aligned}$$

The sensitivity coefficients are :

$$\begin{aligned}c_1 &\equiv \frac{\partial V_{DMM}}{\partial V_{STD}} \quad \dots(4) \\ &= 1\end{aligned}$$

$$\begin{aligned}c_2 &\equiv \frac{\partial V_{DMM}}{\partial \Delta V_{DMM}} \quad \dots(5) \\ &= 1\end{aligned}$$

The combined standard uncertainty becomes :

$$\begin{aligned}u_c^2(V) &= u_1^2(V) + u_2^2(V) \\ u_c(V) &= \sqrt{u_1^2(V) + u_2^2(V)} \quad \dots(6)\end{aligned}$$

4.1 TYPE A EVALUATION

This is a case where the reference standard's accuracy is much better than the device under test's. The multimeter's reading may remain unchanged, or sometimes the multimeter has a ± 1 count flickering due to the multimeter's digitizing process. In this case, the Type A evaluation of standard uncertainty can be assumed negligible and the repeatability uncertainty can be treated as Type B uncertainty using the resolution error of the multimeter.

4.2 TYPE B EVALUATION

4.2.1 Calibrator

From the calibrator's specification, the uncertainty of the applied voltage is :

$$\begin{aligned}a_1 &= 5 \text{ ppm of output} + 4 \text{ } \mu\text{V} \quad \dots(7) \\ &= 5 \times 10^{-6} \times 10 \text{ V} + 4 \times 10^{-6} \text{ V} \\ &= 54 \times 10^{-6} \text{ V}\end{aligned}$$

Given level of confidence is 99 %. Assume normal distribution, coverage factor $k = 2.58$, the standard uncertainty of applied voltage is :

EXAMPLE 5 : CALIBRATION OF DC 20 V RANGE OF A MULTIMETER

$$\begin{aligned}
 u_1(V) &= \frac{a_1}{2.58} && \dots(8) \\
 &= \frac{54 \times 10^{-6}}{2.58} \\
 &= 20.9 \times 10^{-6} \text{ V}
 \end{aligned}$$

Degrees of freedom : $\nu_1 = \infty$

4.2.2 Multimeter

From multimeter's specifications, the 20 V range resolution is 100 μ V (i.e. 1 count). Since the reading was unchanged, assumed the limit is half a count.

$$\begin{aligned}
 a_2 &= \frac{100 \text{ } \mu\text{V}}{2} && \dots(9) \\
 &= 50 \times 10^{-6} \text{ V}
 \end{aligned}$$

Assumed rectangular distribution, the standard uncertainty due to the resolution uncertainty of the multimeter is :

$$\begin{aligned}
 u_2(V) &= \frac{a_2}{\sqrt{3}} && \dots(10) \\
 &= \frac{50 \times 10^{-6}}{\sqrt{3}} \\
 &= 28.9 \times 10^{-6} \text{ V}
 \end{aligned}$$

Degrees of freedom : $\nu_2 = \infty$

5.0 UNCERTAINTY BUDGET

Sources of Uncertainty	Symbol u_i	Type	Uncertainty Value	Probability Distribution	Coverage Factor k	Standard Uncertainty $u(x_i)$ (V)	Sensitivity Coefficient c_i	$ c_i \times u(x_i)$ (V)	Degrees of Freedom ν
Calibrator	$u_1(V)$	B	$54 \times 10^{-6} \text{ V}$	Normal	2.58	20.9×10^{-6}	1	20.9×10^{-6}	∞
Resolution	$u_2(V)$	B	$50 \times 10^{-6} \text{ V}$	Rectangular	$\sqrt{3}$	28.9×10^{-6}	1	28.9×10^{-6}	∞

EXAMPLE 5 : CALIBRATION OF DC 20 V RANGE OF A MULTIMETER

6.0 COMBINED STANDARD UNCERTAINTY $u_c(y)$

The combined standard uncertainty of the indicated voltage at the multimeter is :

$$\begin{aligned}u_c(V) &= \sqrt{u_1^2(V) + u_2^2(V)} && \dots(11) \\&= \sqrt{(20.9 \times 10^{-6})^2 + (28.9 \times 10^{-6})^2} \\&= 35.7 \times 10^{-6} \text{ V}\end{aligned}$$

7.0 EFFECTIVE DEGREES OF FREEDOM ν_{eff}

Since ν_1 and ν_2 are infinite, effective degrees of freedom $\nu_{eff} = \infty$

$$\begin{aligned}\nu_{eff} &= \frac{u_c^4(V)}{\frac{[c_1 u_1(V)]^4}{\nu_1} + \frac{[c_2 u_2(V)]^4}{\nu_2}} && \dots(12) \\&= \frac{(3.57 \times 10^{-6})^4}{\frac{(1 \times 20.9 \times 10^{-6})^4}{\infty} + \frac{(1 \times 28.9 \times 10^{-6})^4}{\infty}} \\&\approx \infty\end{aligned}$$

8.0 EXPANDED UNCERTAINTY, U

For effective degrees of freedom $\nu_{eff} = \infty$, $k = 2$ is used at approximately 95 % level of confidence. Therefore, the expanded uncertainty is

$$\begin{aligned}U &= k u_c(V) && \dots(13) \\&= 2 \times 35.7 \times 10^{-6} \\&= 71 \mu\text{V}\end{aligned}$$

9.0 REPORTING OF RESULTS

$$V_{DMM} = 10.0001 \text{ V} \pm 71 \mu\text{V} \quad \dots(14)$$

The measured result is 10.0001 V. The reported expanded uncertainty is $\pm 71 \mu\text{V}$ with a coverage factor of $k = 2$, assuming a normal distribution at a level of confidence of approximately 95 %.

EXAMPLE 6 : HARMONIC MEASUREMENT

1.0 INTRODUCTION

The measurement configuration is shown in Figure 1. An attenuator is used to improve the mismatch uncertainty.

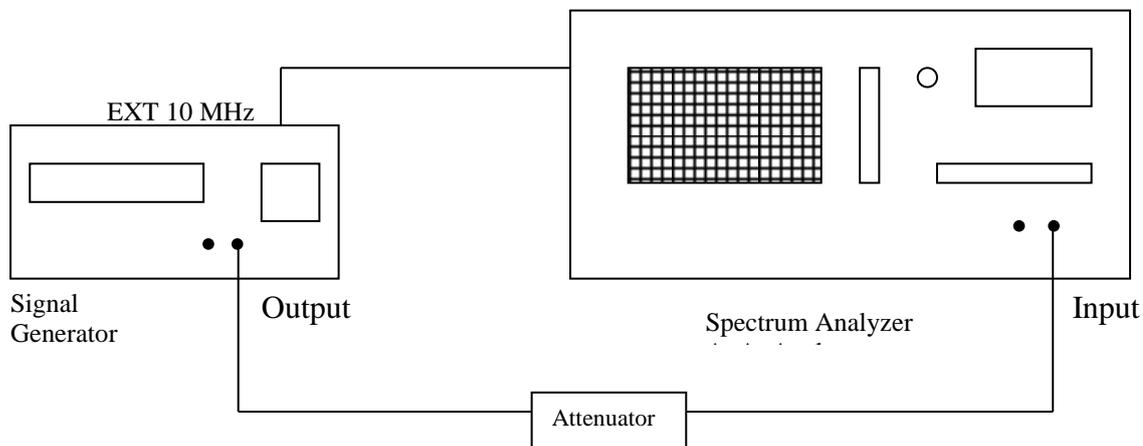


Figure 1: Measurement set-up

The following information is derived from the instrument's specification sheet.

Signal Generator (UUT) SWR	= 2.0
Spectrum Analyzer SWR	= 1.9
Spectrum analyzer instrument error	= 1.72dB
Attenuation	= 10dB
Attenuator flatness	= 1.2 dB
Cable flatness	= 0.5 dB

The harmonic measurement data obtained from the spectrum analyzer are shown in Table 1.

Table 1

No. of Readings	Data
1	40.02
2	40.08
3	39.96
4	40.04
5	39.90

The mean of 5 harmonic measurements is 40 dBc.

EXAMPLE 6 : HARMONIC MEASUREMENT

1.1 The Measurand

The measurand, Harmonic is the power difference between the CW carrier and a signal at a specified offset from the carrier. The offset is a multiple of the carrier frequency.

2.0 MEASUREMENT EQUATION

The measurement equation is:

$$Harmonic, Har = \frac{\left(\frac{Offset_power}{Att_A} \right)}{\left(\frac{Carrier_power}{Att_B} \right)} \times (1 + E_{flatness} + SA_{error})$$

where,

- Offset _ power* : measured level of the harmonic signal (mV).
- Carrier _ power* : measured level of the carrier Level (mW).
- Att_A* : path loss between the signal generator output and the spectrum analyzer input while measuring the harmonic signal, including mismatch.
- Att_B* : path loss between the signal generator output and the spectrum analyzer input while measuring the carrier (reference) signal, including mismatch.
- E_{flatness}* : variation in path loss of the cables, attenuator and adapters (not including SWR) when measuring the harmonic and reference signal as a fraction.
- SA_{error}* : error introduced by the spectrum analyzer as a fraction.

3.0 DERIVATION OF SENSITIVITY COEFFICIENT

$$Harmonic, Har = f(Carrier_power, Offset_power, Att_A, Att_B, E_{flatness}, SA_{error})$$

$$c_1 = \frac{\partial Har}{\partial Carrier_power} \qquad c_2 = \frac{\partial Har}{\partial Offset_power}$$

$$c_3 = \frac{\partial Har}{\partial Att_A} \qquad c_4 = \frac{\partial Har}{\partial Att_B}$$

$$c_5 = \frac{\partial Har}{\partial E_{flatness}} \qquad c_6 = \frac{\partial Har}{\partial SA_{error}}$$

EXAMPLE 6 : HARMONIC MEASUREMENT

c_1 and c_2 do not need to be computed as the *Carrier _ power* and *Offset _ power* are simply the digital readings from the spectrum analyzer. The digital readings have no uncertainty associated with them. Instrumentation errors are contained in SA_{error} .

$$c_3 = \frac{-Offset_power \times (1 + E_{flatness} + SA_{error})}{Carrier_power \times \frac{1}{Att_B} \times Att_A^2}$$

$$= \frac{-Offset_power}{Carrier_power \times Att_A}$$

Note : The above is obtained by assuming Att_A equal to Att_B . The second and third terms of the equation are negligible comparing to the first term and can be ignored.

Similarly,

$$c_4 = \frac{Offset_power}{Carrier_power \times Att_A}$$

$$c_5 = c_6$$

$$= \frac{Offset_power \times Att_B}{Carrier_power \times Att_A}$$

$$= \frac{Offset_power}{Carrier_power}$$

4.0 UNCERTAINTY EQUATION

The uncertainty equation is :

$$u(Har) = \sqrt{[c_3 \times u(Att_A)]^2 + [c_4 \times u(Att_B)]^2 + [c_5 \times u(E_{flatness})]^2 + [c_6 \times u(SA_{error})]^2}$$

$$= \frac{Offset_power}{Carrier_power} \times \sqrt{\left[\frac{u(Att_A)}{Att_A}\right]^2 + \left[\frac{u(Att_B)}{Att_A}\right]^2 + u^2(Cable_flatness) + u^2(SA_{error})}$$

$$= \frac{Offset_power}{Carrier_power} \times \sqrt{2 \left[\frac{u(Att_A)}{Att_A}\right]^2 + u^2(Cable_flatness) + u^2(SA_{error})}$$

where $u(x)$ is the uncertainty of variable x .

EXAMPLE 6 : HARMONIC MEASUREMENT

5.0 TYPE A UNCERTAINTY

Five readings were taken for the harmonic measurement. They are 40.02 dBc, 40.08 dBc, 39.96 dBc, 40.04 dBc and 39.90 dBc.

The mean is

$$\begin{aligned} \bar{x} &= \frac{1}{5} \sum_{i=1}^5 x_i \\ &= 40 \text{ dBc} \end{aligned}$$

The variance is

$$\frac{1}{5-1} \sum_{i=1}^5 (x_i - \bar{x})^2 = 0.005 \text{ dB}^2$$

The standard uncertainty,

$$\begin{aligned} u_A(Har) &= \sqrt{\frac{0.005}{5}} \text{ dB} \\ &= 0.0316 \text{ dB} \end{aligned}$$

6.0 TYPE B UNCERTAINTY DERIVATION

6.1 $u(Att_A)$ and $u(Att_B)$ Derivation

The network consisting of the UUT, attenuator and spectrum analyzer can be represented in a signal flowgraph as below:

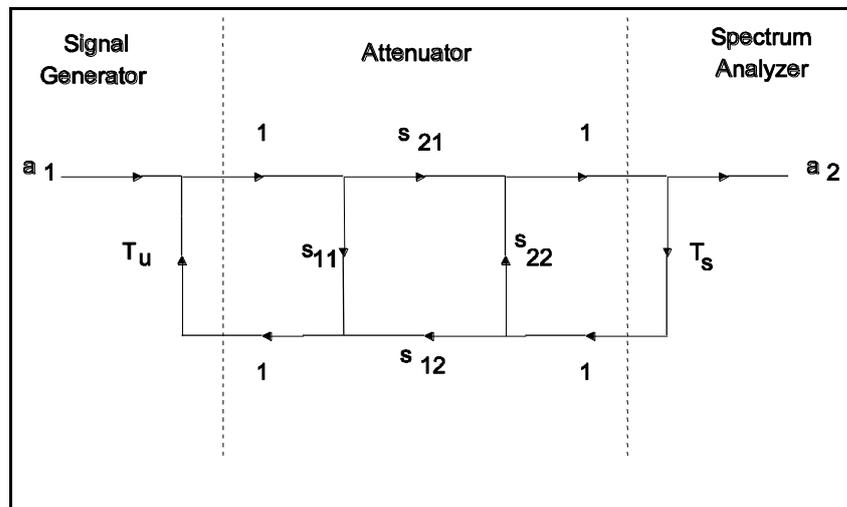


Figure 6.1: Signal flow-graph representation of the measurement network

EXAMPLE 6 : HARMONIC MEASUREMENT

T_u and T_s are the reflection coefficient of the signal generator and spectrum analyzer respectively.

Using Mason's rule, the ratio $\frac{a_2}{a_1}$ can be expressed:

$$\frac{a_2}{a_1} = \frac{s_{21}}{1 - (T_u s_{11} + s_{22} T_s + s_{21} s_{12} T_u T_s) + (T_u s_{11})(T_s s_{22})}$$

$$\approx \frac{s_{21}}{1 - (T_u s_{11} + T_s s_{22} + s_{12} s_{21} T_u T_s)}$$

This formula determines the attenuation between a_1 and a_2 . The minimum and maximum attenuation is given by:

$$Att_{\min} = \frac{s_{21}}{1 + (|T_u| |s_{11}| + |T_s| |s_{22}| + |T_u| |T_s| |s_{21}| |s_{12}|)}$$

$$Att_{\max} = \frac{s_{21}}{1 - (|T_u| |s_{11}| + |T_s| |s_{22}| + |T_u| |T_s| |s_{21}| |s_{12}|)}$$

The above assumed no phase information and constitute the worst case attenuation.

For a coaxial attenuator,

$$s_{21} = s_{12}$$

$$= 10^{\left(\frac{-Att(dB)}{20}\right)}$$

This modifies the minimum and maximum attenuation equations to become:

$$Att_{\min} = \frac{10^{\left(\frac{-Att(dB)}{20}\right)}}{1 + \left[|T_u| |s_{11}| + |T_s| |s_{22}| + 10^{\left(\frac{-Att(dB)}{10}\right)} |T_u| |T_s| \right]}$$

$$Att_{\max} = \frac{10^{\left(\frac{-Att(dB)}{20}\right)}}{1 - \left[|T_u| |s_{11}| + |T_s| |s_{22}| + 10^{\left(\frac{-Att(dB)}{10}\right)} |T_u| |T_s| \right]}$$

EXAMPLE 6 : HARMONIC MEASUREMENT

The maximum and minimum attenuation equations are linear and include uncertainty caused by mismatch. The attenuation Att_{max} , Att_{min} and s_{21} are squared to convert the attenuation into power form. The power attenuation uncertainty is given by:

$$u(Att) = \frac{(Att_{min/max})^2 - s_{21}^2}{s_{21}^2}$$

$$u(Att) = \left[\frac{(Att_{min/max})^2}{s_{21}^2} - 1 \right]$$

$$= \left\{ \frac{1}{\left[1 \pm (|T_u| |s_{11}|) + |T_s| |s_{22}| + 10^{-\left(\frac{Att(dB)}{10}\right)} |T_s| |T_u| \right]^2} - 1 \right\}$$

Hence, for a unit under test SWR of 2.0 : 1, spectrum analyzer's SWR of 1.9 : 1 and attenuator's SWR (s_{11} and s_{22}) of 1.015 : 1, the uncertainty is computed as :

$$|T_u| = \frac{2-1}{2+1}$$

$$= 0.3333$$

$$|T_s| = \frac{1.9-1}{1.9+1}$$

$$= 0.3103$$

$$|s_{11}| = |s_{22}|$$

$$= \frac{1.015-1}{1.015+1}$$

$$= 0.0075$$

$$u(Att) = \left\{ \frac{1}{\left[1 \pm \left(0.3333 \times 0.0075 + 0.3103 \times 0.0075 + 10^{-\left(\frac{10}{10}\right)} \times 0.3103 \times 0.3333 \right) \right]^2} - 1 \right\}$$

$$u(Att) = 3.1 \%, \quad -2.9 \%$$

EXAMPLE 6 : HARMONIC MEASUREMENT

This corresponds to a $10 \times \log(1.031) = 0.13$ dB uncertainty due to mismatch. To convert this uncertainty into the standard derivation form, this value has to be divided by a factor of $\sqrt{2}$. This gives a standard uncertainty of :

$$u(Att_A) = \frac{0.031}{\sqrt{2}}$$

$$= 0.02192 \frac{Watt}{Watt}$$

6.2 $u(E_{flatness})$ Derivation

The uncertainty is contributed by the cable and attenuation flatness. The worst case will be the sum of the two terms. To obtain the standard uncertainty, a normalise factor of $\frac{1}{\sqrt{3}}$ is used. The uncertainty is:

$$u(E_{flatness}) = \frac{10^{\left(\frac{1.2 dB + 0.5 dB}{10}\right)} - 1}{\sqrt{3}}$$

$$= 0.2766$$

6.3 $u(SA_{error})$ Derivation

The spectrum analyzer uncertainty is given as 1.72dB. The standard uncertainty will be 1.72dB multiple by a normalise factor of $\frac{1}{\sqrt{3}}$.

$$u(SA_{error}) = \frac{10^{\left(\frac{1.72 dB}{10}\right)} - 1}{\sqrt{3}}$$

$$= 0.2805$$

6.4 Overall Type B uncertainty

The measured harmonic is 40dBc. In linear form , this is equal to

$$\frac{Offset_power}{Carrier_power} = 10^{\left(\frac{-40 dB}{10}\right)}$$

$$= 1 \times 10^{-4} \frac{Watt}{Watt}$$

$$u(Har) = \frac{Offset_power}{Carrier_power} \times \sqrt{\left[\left(\frac{u(Att_A)}{Att_A}\right)^2 + \left(\frac{u(Att_B)}{Att_A}\right)^2 + u^2(Cable_{flatness}) + u^2(SA_{error})\right]}$$

$$= 1 \times 10^{-4} \times \sqrt{\left(\frac{0.02192}{10}\right)^2 + \left(\frac{0.02192}{10}\right)^2 + (0.2766)^2 + (0.2805)^2}$$

$$= 3.9395 \times 10^{-5}$$

EXAMPLE 6 : HARMONIC MEASUREMENT

Convert to dB, we have the uncertainty as:

$$10 \times \log (1 \times 10^{-4} + 3.9395 \times 10^{-5}) + 40 \text{ dB} = 1.442 \text{ dB}$$

7.0 UNCERTAINTY BUDGET TABLE

Sources of Uncertainty	Symbol x_i	Type	Uncertainty Value	Probability Distribution	Coverage Factor	Standard Uncertainty $u(x_i)$	Sensitivity Coefficient c_i	$ c_i \times u(x_i)$	Degrees of freedom
Attenuator A (include mismatch)	$u(Att_A)$	B	0.031	U-shape	$\sqrt{2}$	0.02192	10^{-5}	0.02×10^{-5}	∞
Attenuator B (include mismatch)	$u(Att_B)$	B	0.031	U-shape	$\sqrt{2}$	0.02192	10^{-5}	0.02×10^{-5}	∞
Spectrum Analyzer	$u(SA_{Error})$	B	0.4859	Rectangular	$\sqrt{3}$	0.2805	10^{-4}	2.8×10^{-5}	∞
Path Variation	$u(E_{flatness})$	B	0.4791	Rectangular	$\sqrt{3}$	0.2766	10^{-4}	2.76×10^{-5}	∞
Repeatability	$u_A(Har)$	A	-	-	-	0.0316 dB	1		4

8.0 COMBINED STANDARD UNCERTAINTY

The combined uncertainty,

$$\begin{aligned} u_c(Har) &= \sqrt{u_A^2(Har) + u_B^2(Har)} \\ &= \sqrt{0.0316^2 + 1.442^2} \\ &= 1.442 \text{ dB} \end{aligned}$$

9.0 EFFECTIVE DEGREES OF FREEDOM v_{eff}

The effective degrees of freedom will be

$$\begin{aligned} v_{eff} &= \frac{u_c^4(Har)}{\frac{u_A^4(Har)}{v_A} + \frac{u_B^4(Har)}{v_B}} \\ &= \frac{1.442^4}{\frac{0.0316^4}{4} + \frac{1.442^4}{\infty}} \\ &\approx \infty \end{aligned}$$

EXAMPLE 6 : HARMONIC MEASUREMENT

10.0 EXPANDED UNCERTAINTY

For an effective degrees of freedom $\nu_{eff} = \infty$, $k = 2$ is used at approximately 95 % level of confidence. Hence, the expanded uncertainty

$$\begin{aligned}U &= k u_c(Har) \\ &= 2 \times 1.442 \\ &= 2.884 \text{ dB}\end{aligned}$$

11.0 REPORTING OF RESULT

The measured harmonic is 40 dBc. The expanded uncertainty of the measurement is ± 2.884 dB at a confidence level of approximately 95 % with a coverage factor of $k = 2$.

EXAMPLE 7: POWER REFERENCE OUTPUT TEST

The thermistor mount when connected to the Power reference output of the Power Meter produces V_{comp} and V_{RF} (V_0 and V_1). A multimeter is used to measure the voltages V_{comp} and V_{RF} (V_0 and V_1).

2.1 Type A Uncertainty Evaluation, $u_A(P_{RF})$

In this test the power reference is obtained indirectly using the formula above. Therefore the measurements made from the multimeter is used to evaluate the uncertainty.

The following steps are taken to ensure accurate measurement :

- 1) The voltage measuring terminals are at the power meter; therefore it is located in an area of free moving air where it is not affected by local heat source.
- 2) The power meter is then stabilized at its operating temperature.
- 3) The multimeter is properly set up according to the operating manual to achieve optimum conditions and it is stabilized at its operating temperature.
- 4) The voltage values are measured, taking five measurements with a complete breakdown between them.

Table of readings

measurement (q)	R	V_{comp}	V_0	V_1	P_{RF} (calculated)
1	200.27 Ω	5.15 V	71.67 μ V	78.35 mV	P_{RF1}
2	200.28 Ω	5.15 V	93.60 μ V	78.37 mV	P_{RF2}
3	200.27 Ω	5.14 V	75.67 μ V	78.28 mV	P_{RF3}
4	200.27 Ω	5.15 V	83.25 μ V	78.34 mV	P_{RF4}
5	200.27 Ω	5.16 V	79.67 μ V	78.35 mV	P_{RF5}

$$\begin{aligned}\text{Mean of } P_{RF} &= \bar{q} \\ &= \frac{1}{5} \sum_{k=1}^5 q_k \\ &= \frac{(P_{RF1} + P_{RF2} + P_{RF3} + P_{RF4} + P_{RF5})}{5} \\ &= \frac{(0.99999 + 0.99998 + 0.99999 + 1.0 + 0.99998) \times 10^{-3}}{5} \\ &= 0.99998 \times 10^{-3} \text{ Watts}\end{aligned}$$

EXAMPLE 7: POWER REFERENCE OUTPUT TEST

$$\begin{aligned}\text{Experimental standard deviation} &= s(q_k) \\ &= \sqrt{\frac{1}{4} \sum_{k=1}^5 (q_k - \bar{q})^2} \\ &= 8.3667 \times 10^{-9} \text{ Watts}\end{aligned}$$

$$\begin{aligned}\text{Experimental SD of the mean} &= \frac{s(q_k)}{\sqrt{5}} \\ &= \frac{(8.3667 \times 10^{-9})}{\sqrt{5}} \\ &= 3.742 \times 10^{-9} \text{ Watts} \\ &= u_A(P_{RF})\end{aligned}$$

2.2 Type B Uncertainty Evaluation, $u_B(P_{RF})$

2.2.1 Power Meter Accuracy

$\pm 1\%$ of full scale on all ranges (+ 0 °C to 55 °C)

2.2.2 Thermistor Mount

From calibration report:

12 months calibration cycle

Frequency : 50 MHz

Serial No : 84226

Calibration Factor = 99.92 % \pm 0.34 %

The $\pm 0.34\%$ is the uncertainty value dCF of the calibration factor CF

Effective Calibration Factor uncertainty : $0.9992 \times 0.0034 = \mathbf{0.0034}$

dCF = 0.0034

Taking Rectangular distribution:

$$\begin{aligned}\text{Standard uncertainty } y, u_5(CF) &= \frac{0.0034}{\sqrt{3}} \\ &= 0.002\end{aligned}$$

From measurement :

Thermistor Mount internal bridge resistance : 200 Ω

The resistance is measured from the multimeter, range of 1 k Ω .

Standard uncertainty $y, u_4(R) = 8.1 \text{ m}\Omega$

This is the measurement uncertainty of the multimeter in the range 1 kW when measuring resistance where we consider a worst case of the measurement being done on the 90th day after calibration date to include a worst case (calibration interval 90 days)

EXAMPLE 7: POWER REFERENCE OUTPUT TEST

2.2.3 Multimeter

Accuracy : ppm of Reading + ppm of Range

Range	1 Year
100 mV	9 + 3
1 V	8 + 0.3
10 V	8 + 0.05
100 V	10 + 0.3
1000 V	10 + 0.1

Multimeter Specification

Voltage	Reading	Range	Uncertainty
V_0	0.024 mV	100 mV	0.3 μ V
V_1	81.07 mV	100 mV	1.03 μ V
V_{comp}	4.98 V	10 V	0.041 mV

Symbol	Uncertainty	Standard Uncertainty
$u_1(V_0)$	0.3 μ V	173.2 nV
$u_2(V_1)$	1.03 μ V	0.59 μ V
$u_3(V_{comp})$	0.041 mV	0.024 mV

Taking 95 % confidence interval with coverage factor $k = 2$.
Assuming rectangular distribution.

2.2.4 Power Reference Output Power Uncertainty Due To Mismatch Uncertainty of power meter and thermistor mount Reflection Coefficient

Power Reference Output SWR

SWR : 1.05

$$\text{Reflection Coefficient : } \frac{SWR - 1}{SWR + 1}$$

$$= \underline{\underline{0.0244}} \text{ (From specifications)}$$

EXAMPLE 7: POWER REFERENCE OUTPUT TEST

Thermistor Mount

Worst case reflection coefficient is 0.007 (Sum of mismatch and mismatch error = 0.001 + 0.006)

Mismatch thermistor mount to Power Reference Output in log scale:

$$20 \text{ Log}(1 + 0.0244 \times 0.007) = 0.001483$$

$$20 \text{ Log}(1 - 0.0244 \times 0.007) = -0.001483$$

$$\begin{aligned} \text{Power Uncertainty due to mismatch} &= 1 \text{ mW} - (10^{\frac{0.0014}{10}} \times 1 \text{ mW}) \\ &= 3.414 \times 10^{-7} \text{ W} \end{aligned}$$

$$\begin{aligned} \text{Standard uncertainty } y &= \frac{3.414 \times 10^{-7} \text{ W}}{\sqrt{2}} \\ &= 2.414 \text{ nW} \end{aligned}$$

Assuming U-Shaped distribution

Assuming Sensitivity Coefficient equal to 1. $c_6 = 1$.

3.0 MEASUREMENT UNCERTAINTY BUDGET

Formula given :

$$P_{RF} = \frac{2V_{comp}(V_1 - V_o) + V_o^2 - V_1^2}{4R(\text{CALIBRATION_FACTOR})}$$

From the formula, the power reference level is dependent on a few variables. They are V_o , V_1 , V_{comp} , R and Calibration Factor. We have to analyze the effect of each variable to the measured power reference level.

The best way to analyze the effect of each variable deviation on power reference level is to perform partial differentiation. Differentiate P_{RF} with respect to one variable at a time, then Root-Sum-Square all uncertainties to calculate the combined standard uncertainty.

$$P_{RF} = \frac{2V_{comp}(V_1 - V_o) + V_o^2 - V_1^2}{4R(\text{CALIBRATION_FACTOR})}$$

EXAMPLE 7: POWER REFERENCE OUTPUT TEST

4.0 SENSITIVITY COEFFICIENTS

$$\begin{aligned}\frac{\partial P_{RF}}{\partial V_0} &= \frac{-V_{comp} + V_0}{2R(CALIBRATION_FACTOR)} \\ &= \frac{-4.98 \text{ V} + 0.024 \text{ mV}}{2(200)(0.9992)} \\ &= -12.55 \times 10^{-3} \text{ A}\end{aligned}$$

$$\begin{aligned}\frac{\partial P_{RF}}{\partial V_1} &= \frac{V_{comp} - V_1}{2R(CALIBRATION_FACTOR)} \\ &= \frac{4.98 \text{ V} - 81.07 \text{ mV}}{2(200)(0.9992)} \\ &= 12.26 \times 10^{-3} \text{ A}\end{aligned}$$

$$\begin{aligned}\frac{\partial P_{RF}}{\partial V_{comp}} &= \frac{V_1 - V_0}{2R(CALIBRATION_FACTOR)} \\ &= \frac{81.07 \text{ mV} - 0.024 \text{ mV}}{2(200)(0.9992)} \\ &= 202.78 \times 10^{-3} \text{ A}\end{aligned}$$

$$\begin{aligned}\frac{\partial P_{RF}}{\partial R} &= \frac{-[2V_{comp}(V_1 - V_0)] + V_0^2 - V_1^2}{4R^2(CALIBRATION_FACTOR)} \\ &= \frac{-[2(4.98)(81.07 \times 10^{-3} \text{ V} - 0.024 \times 10^{-3} \text{ V})] + (0.024 \times 10^{-3} \text{ V})^2 - (81.07 \times 10^{-3} \text{ V})^2}{4(200)^2(0.9992)} \\ &= -5.01 \times 10^{-6} \text{ A}\end{aligned}$$

$$\begin{aligned}\frac{\partial P_{RF}}{\partial CF} &= \frac{-[2V_{comp}(V_1 - V_0)] + V_0^2 - V_1^2}{4R^2(CALIBRATION_FACTOR)} \\ &= \frac{-[2(4.98)(81.07 \times 10^{-3} \text{ V} - 0.024 \times 10^{-3} \text{ V})] + (0.024 \times 10^{-3} \text{ V})^2 - (81.07 \times 10^{-3} \text{ V})^2}{4(200)(0.9992)^2} \\ &= -1.00 \times 10^{-3} \text{ W} \quad (\text{Power} = VI)\end{aligned}$$

(CF abbreviation for CALIBRATION_FACTOR)

EXAMPLE 7: POWER REFERENCE OUTPUT TEST

$$\text{Let } c_1 = \frac{\partial P_{RF}}{\partial V_o}, c_2 = \frac{\partial P_{RF}}{\partial V_1}, c_3 = \frac{\partial P_{RF}}{\partial V_{comp}}, c_4 = \frac{\partial P_{RF}}{\partial R}, c_5 = \frac{\partial P_{RF}}{\partial CF}, c_6 = 1$$

Where c_1, c_2, c_3, c_4, c_5 and c_6 are Sensitivity Coefficients

5.0 UNCERTAINTY BUDGET TABLE

Source of Uncertainty	Symbol X_i	Type	Uncertainty value	Probability distribution	Coverage factor	Standard uncertainty $u(x_i)$	Sensitivity coefficient c_i	$ c_i \times u(x_i)$ (W)	Degrees of freedom ν
V_o of power meter	$u_1(V)$	B	0.3 μ V	Rectangular	$\sqrt{3}$	173.2×10^{-9}	-12.55×10^{-3}	2×10^{-9}	∞
V_1 of power meter	$u_2(V)$	B	1.03 μ V	Rectangular	$\sqrt{3}$	0.59×10^{-6}	12.26×10^{-3}	7×10^{-9}	∞
V_{comp} of power meter	$u_3(V)$	B	0.041 mV	Rectangular	$\sqrt{3}$	0.024×10^{-3}	202.78×10^{-6}	4×10^{-9}	∞
Thermistor Mount Internal bridge Resistance	$u_4(R)$	B	-	Rectangular	$\sqrt{3}$	8.1×10^{-3}	-5.01×10^{-6}	40×10^{-9}	∞
Thermistor Mount Calibration Factor	$u_5(CF)$	B	0.0034	Rectangular	$\sqrt{3}$	0.002	-1.00×10^{-3}	0.000002	∞
Power Mismatch	$u_6(W)$	B	3.41×10^{-7} Watts	U-shaped	$\sqrt{2}$	2.41×10^{-9}	1	2.41×10^{-9}	∞

6.0 COMBINED STANDARD UNCERTAINTY

$$u_c^2(P_{RF}) = u_A^2(P_{RF}) + u_B^2(P_{RF})$$

Type A uncertainty is negligible and hence ignored.

$$u_B^2(P_{RF}) = \sum_{i=1}^6 (c_i)^2 u_i(x_i)$$

$u_1(x_1) = u_1(V) =$ Standard Uncertainty of voltage V_o .

$u_2(x_2) = u_2(V) =$ Standard Uncertainty of voltage V_1 .

$u_3(x_3) = u_3(V) =$ Standard Uncertainty of voltage V_{comp} .

$u_4(x_4) = u_4(R) =$ Standard Uncertainty of Thermistor Mount internal bridge resistance.

$u_5(x_5) = u_5(CF) =$ Standard Uncertainty of Thermistor Mount Calibration Factor.

$u_6(x_6) = u_6(W) =$ Standard Power Uncertainty due to mismatch.

EXAMPLE 7: POWER REFERENCE OUTPUT TEST

$$\begin{aligned}u_B^2(P_{RF}) &= (c_1)^2 u_1^2(V) + (c_2)^2 u_2^2(V) + (c_3)^2 u_3^2(V) + (c_4)^2 u_4^2(R) + (c_5)^2 u_5^2(CF) + (c_6)^2 u_6^2(W) \\&= (-12.55 \times 10^{-3})^2 (173.2 \times 10^{-9})^2 + (12.26 \times 10^{-3})^2 (0.59 \times 10^{-6})^2 \\&\quad + (202.78 \times 10^{-6})^2 (0.024 \times 10^{-3})^2 + (-5.01 \times 10^{-6})^2 (8.1 \times 10^{-3})^2 \\&\quad + (-1.00 \times 10^{-3})^2 (0.002)^2 + (1)^2 (2.414 \times 10^{-9})^2 \\&= 4.003 \times 10^{-12} \\u_B(P_{RF}) &= 2.0 \times 10^{-6}\end{aligned}$$

Therefore,

$$\begin{aligned}u_c(P_{RF}) &= \sqrt{u_A^2(P_{RF}) + u_B^2(P_{RF})} \\&= 2.0 \mu\text{W}\end{aligned}$$

7.0 EFFECTIVE DEGREES OF FREEDOM ν_{eff}

The effective degrees of freedom,

$$\begin{aligned}\nu_{eff} &= \frac{u_c^4(P_{RF})}{\frac{u_A^4(P_{RF})}{\nu_A} + \frac{u_B^4(P_{RF})}{\nu_B}} \\&= \frac{(2.0 \times 10^{-6})^4}{\frac{(3.742 \times 10^{-9})^4}{4} + \frac{(2.0 \times 10^{-6})^4}{\infty}} \\&= \infty\end{aligned}$$

8.0 EXPANDED UNCERTAINTY

For an effective degrees of freedom $\nu_{eff} = \infty$, $k = 2$ is used at approximately 95 % level of confidence. Therefore, the expanded uncertainty

$$\begin{aligned}U &= k u_c(P_{RF}) \\&= 2 \times 2.0 \mu\text{W} \\&= 4.0 \mu\text{W}\end{aligned}$$

9.0 REPORTING OF RESULT

The power reference of the power meter is specified as 1 mW. The expanded uncertainty of measurement is $\pm 4 \mu\text{W}$ at a confidence level of approximately 95 % with a coverage factor of $k = 2$.

EXAMPLE 9: CALIBRATION OF AN INDUSTRIAL PRESSURE GUAGE

1. INTRODUCTION

The transition time is normally measured at 10% to 90% of amplitude. To achieve this measurement on a digital oscilloscope, a programmable 500 MHz Pulse Generator whereby all output signals have a fixed transition time ≤ 200 ps is used. The Pulse Generator then applies a fast rise time pulse to the Digitizing Oscilloscope and makes an automatic rise-time measurement.

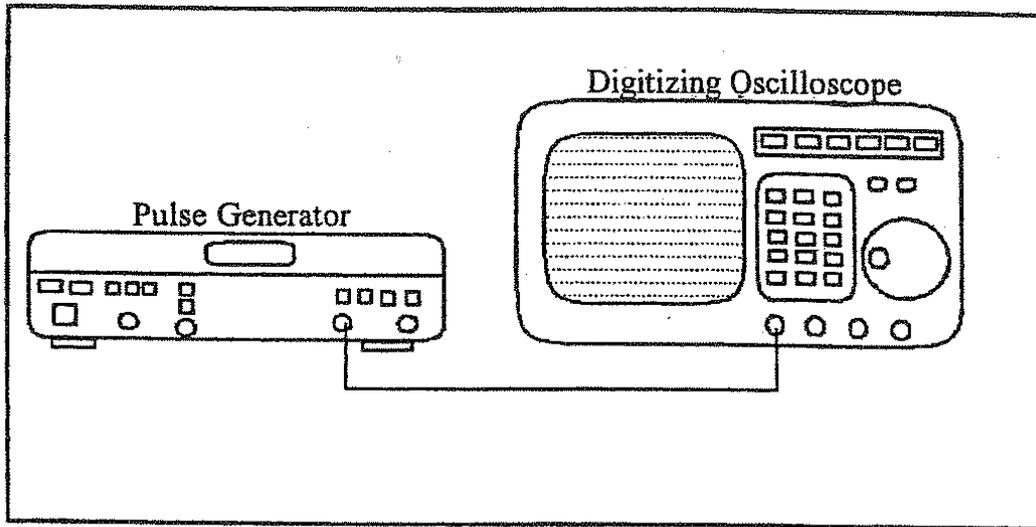


Figure 1: Measurement set-up

1.1 Equipment Used:

500 MHz Pulse Generator

1.2 Specifications:

The Digitizing Oscilloscope has a repetitive transition time that is ≤ 700 ps

2.0 MEASUREMENT UNCERTAINTY ANALYSIS

To calculate the Rise-Time of the signal observed in the scope:

$$RT_{Observed} = \sqrt{RT_{Scope}^2 + RT_{Gen}^2}$$

where,

$RT_{Observed}$: Rise-time of signal observed on oscilloscope

RT_{Scope} : Rise-time of the oscilloscope itself

RT_{Gen} : Rise-time of the Pulse of the Pulse Generator

Since we are to determine the Rise-time of the Digitizing Oscilloscope, we shall rearrange the equation. Therefore the Measurement Equation we are using for Rise-time of the Scope is :

$$RT_{Scope} = \sqrt{RT_{Observed}^2 - RT_{Gen}^2}$$

EXAMPLE 9: CALIBRATION OF AN INDUSTRIAL PRESSURE GAUGE

There is a reminder to take note of is that the rise-time of the oscilloscope is not to be more than 5 times as fast as the signal rise-time as the displayed rise-time will become slower than actual.

2.1 Type A Uncertainty Evaluation $u_A (RT_{Scope})$

In this test the Rise-time of the oscilloscope is obtained indirectly using the formula above; therefore the measurements made from the oscilloscope are used to evaluate the uncertainty.

The following steps are taken to ensure accurate measurement :

1. The Oscilloscope and Pulse Generator are to be located in an area of free moving air where they are not affected by local heat source.
2. The pulse generator is stabilized at its operating temperature of 0 °C to +55 °C with a 30 minutes warm up phase with a 50 ohms load resistance at all outputs.
3. The oscilloscope is properly set up according to the operating manual to achieve optimum conditions and it is stabilized at its operating temperature +15°C to +55 °C with a 30 minutes Power On warm up.
4. The rise-time values are measured and taking five measurements with a complete breakdown between them.

Table of readings

Measurement (i)	$RT_{Observed}$	RT_{Gen}	RT_{Scope} (Calculated)	RT_{si}
1	540 ps	200 ps	501.60 ps	RT_{s1}
2	539 ps	200 ps	500.52 ps	RT_{s2}
3	540 ps	200 ps	501.60 ps	RT_{s3}
4	540 ps	200 ps	501.60 ps	RT_{s4}
5	539 ps	200 ps	500.52 ps	RT_{s5}

$$\begin{aligned}
 \text{Mean of } RT_{Scope} &= \bar{q} \\
 &= \frac{1}{5} \sum_{k=1}^5 q_k \\
 &= \frac{(RT_{s1} + RT_{s2} + RT_{s3} + RT_{s4} + RT_{s5})}{5} \\
 &= \frac{(501.60 + 500.52 + 501.60 + 501.60 + 500.52)}{5} \\
 &= \frac{2505.84}{5} \\
 &= 501.17 \text{ ps}
 \end{aligned}$$

EXAMPLE 9: CALIBRATION OF AN INDUSTRIAL PRESSURE GAUGE

Experimental standard deviation,

$$\begin{aligned} s(q_k) &= \sqrt{\frac{1}{4} \left[\sum_{k=1}^5 (q_k - \bar{q})^2 \right]} \\ &= 0.59 \text{ ps} \end{aligned}$$

Experimental SD of the mean,

$$\begin{aligned} s\left(\bar{q}_k\right) &= \frac{0.59 \text{ ps}}{\sqrt{5}} \\ &= 0.26 \text{ ps} \\ &= u_A(RT_{Scope}) \end{aligned}$$

2.2 Type B Uncertainty Evaluation $u_B(RT_{Scope})$

2.2.1 Pulse Generator Transition Time Specification

10% to 90% amplitude : ≤ 200 ps, 300 mV to 3V Range

Standard Uncertainty of Pulse Generator,

$$\begin{aligned} u_{Gen} &= \frac{200 \text{ ps}}{\sqrt{3}} \\ &= 115.47 \text{ ps} \end{aligned}$$

(The probability distribution of the rise-time specification is not exactly known. Hence, rectangular distribution is adopted as given in GUM)

2.2.2 The Rise-time of the observed Signal

The reading observed is 539 ps and the uncertainty present here is only the resolution of the displayed reading of the Rise-time value and it is considered negligible.

3.0 MEASUREMENT UNCERTAINTY BUDGET

Formula given :

$$RT_{Scope} = \sqrt{RT_{Observed}^2 - RT_{Gen}^2}$$

From the formula, the Rise-time of scope is now dependent only on the standard uncertainty of the Pulse Generator.

EXAMPLE 9: CALIBRATION OF AN INDUSTRIAL PRESSURE GAUGE

The best way to analyze the effect of the variable deviation on the Rise-time is to perform partial differentiation. Differentiate RT_{Scope} with respect to RT_{Gen} , then Root-Sum-Square the uncertainty to calculate the combined standard uncertainty.

Therefore,

$$RT_{Scope} = \sqrt{RT_{Observed}^2 - RT_{Gen}^2}$$

4.0 SENSITIVITY COEFFICIENTS

Taking partial derivatives of the equation, the sensitivity coefficients

$$\begin{aligned}
 C_1 &= \frac{\partial RT_{Scope}}{\partial RT_{Gen}} \\
 &= \frac{\partial \sqrt{RT_{Observed}^2 - RT_{Gen}^2}}{\partial RT_{Gen}} \\
 &= \frac{-RT_{Gen}}{\sqrt{RT_{Observed}^2 - RT_{Gen}^2}}
 \end{aligned}$$

Substituting $RT_{Observed} = 539$ ps and $RT_{Gen} = 200$ ps

$$\begin{aligned}
 C_1 &= \frac{-200}{\sqrt{539^2 - 200^2}} \\
 &= -0.40
 \end{aligned}$$

5.0 UNCERTAINTY BUDGET TABLE

Source of Uncertainty	Symbol x_i	Type	Uncertainty Value	Probability distribution	Coverage factor	Standard Uncertainty $u(x_i)$	Sensitivity coefficient c_i	$ c_i u(x_i)$	Degrees of freedom
Rise-time of pulse generator	$U_{Gen}(t)$	B	200 ps	Rectangular	$\sqrt{3}$	115.47 ps	-0.40 ps	46.19	∞
Rise time of oscilloscope	$u_B(RT_{Scope})$	B	-	t-distribution	-	46.19 ps			∞

EXAMPLE 9: CALIBRATION OF AN INDUSTRIAL PRESSURE GAUGE

6.0 COMBINED STANDARD UNCERTAINTY

$$u_c^2(RT_{Scope}) = u_A^2(RT_{Scope}) + u_B^2(RT_{Scope})$$

The Type A uncertainty is negligible and hence ignored

$$\begin{aligned} u_B(RT_{Scope}) &= \sqrt{c_1^2 u_{Gen}^2} \\ &= \sqrt{-0.40 \times 115.47} \text{ ps} \\ &= 46.19 \text{ ps} \end{aligned}$$

Hence,

$$\begin{aligned} u_c(RT_{Scope}) &= \sqrt{u_A^2(RT_{Scope}) + u_B^2(RT_{Scope})} \\ &= 46.19 \text{ ps} \end{aligned}$$

7.0 EFFECTIVE DEGREES OF FREEDOM ν_{eff}

The effective degrees of freedom

$$\begin{aligned} \nu_{eff} &= \frac{u_c^4(RT_{Scope})}{\frac{u_A^4(RT_{Scope})}{\nu_A} + \frac{u_B^4(RT_{Scope})}{\nu_B}} \\ &= \frac{(46.19 \times 10^{-12})^4}{\frac{(0.26 \times 10^{-12})^4}{4} + \frac{(46.19 \times 10^{-12})^4}{\infty}} \\ &= \infty \end{aligned}$$

8.0 EXPANDED UNCERTAINTY

For an effective degrees of freedom, $\nu_{eff} = \infty$, $k = 2$ is used at approximately 95% level of confidence. Therefore, the expanded uncertainty

$$\begin{aligned} U &= k u_c(RT_{Scope}) \\ &= 2 \times 46.19 \\ &= 92.38 \text{ ps} \end{aligned}$$

9.0 REPORTING OF RESULT

The rise-time of the digital oscilloscope is specified as ≤ 700 ps. The uncertainty of measurement is ± 92.38 ps at a confidence level of approximately 95% with a coverage factor $k = 2$

EXAMPLE 9: CALIBRATION OF AN INDUSTRIAL PRESSURE GAUGE

1.0 INTRODUCTION

The increasing and decreasing gauge pressure indications of the Industrial Pressure Gauge are calibrated by comparing against a Pressure Test Gauge.

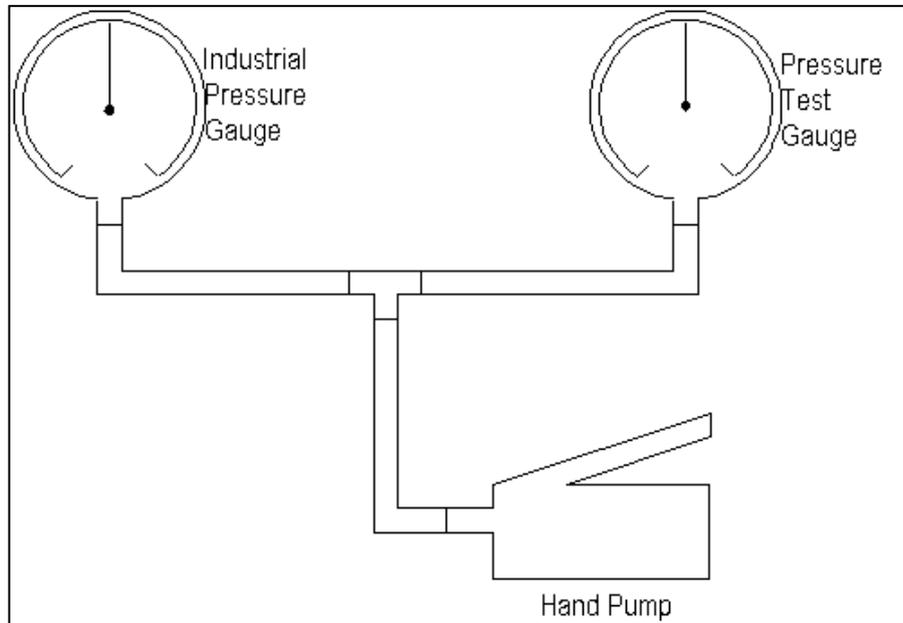


Figure 1: Set-up of an industrial pressure gauge calibration

The unit-under-test (UUT) is an Industrial Pressure Gauge with:

Range : 0 - 100 psi
Scale: 1 div = 1 psi
Resolution: $\frac{1}{2}$ div = 0.5 psi
Accuracy class: 1

It is calibrated against a Pressure Test Gauge with:

Range : 0 - 100 psi
Scale: 1 div = 0.2 psi
Resolution: $\frac{1}{2}$ div = 0.1 psi
Uncertainty: $\pm 0.25\%$ span

EXAMPLE 9: CALIBRATION OF AN INDUSTRIAL PRESSURE GAUGE

2.0 UUT MEASURED DATA

The number of readings for each test point, n = 5, are tabulated as below:

Table 1 - Increasing Gauge Pressure

Nominal Pressure as from Test Gauge	Readings of UUT (psi)					Mean (psi)	Standard Deviation (psi)
	1	2	3	4	5		
20	20.0	20.5	20.0	20.0	20.5	20.2	0.2739
40	40.0	40.5	40.5	40.5	40.0	40.3	0.2739
60	60.5	60.5	60.5	60.5	60.0	60.4	0.2236
80	80.0	80.5	80.5	80.5	80.5	80.4	0.2236
100	100.0	100.0	100.5	100.5	100.5	100.3	0.2739

Table 2 - Decreasing Gauge Pressure

Nominal Pressure as from Test Gauge	Readings of UUT (psi)					Mean (psi)	Standard Deviation (psi)
	1	2	3	4	5		
80	80.0	80.5	80.0	80.0	80.0	80.1	0.2236
60	59.5	59.5	59.5	59.5	60.0	59.6	0.2236
40	40.5	40.0	40.0	40.5	40.5	40.3	0.2739
20	20.0	20.0	20.0	20.5	20.0	20.2	0.2739

3.0 MATHEMATICAL MODEL

Hence the mathematical relationship can be modelled as:

$$Y_{UUT} = X_{Test Gauge} + E_{Rep} + E_{Res} \quad \dots(1)$$

where

Y_{UUT} : Industrial Pressure Gauge indication.

$X_{Test Gauge}$: Pressure Test Gauge indication.

E_{Rep} : Error due to random variations of the Industrial Pressure Gauge indication when repeatedly applied of same Pressure Test Gauge indication.

E_{Res} : Error due to resolution of the Industrial Pressure Gauge indication.

EXAMPLE 9: CALIBRATION OF AN INDUSTRIAL PRESSURE GAUGE

4.0 UNCERTAINTY EQUATION

Considering all the input quantities are uncorrelated, the combined standard uncertainty equation is then given by:

$$u_c(Y_{UUT}) = \sqrt{\left[\left(\frac{\partial Y_{UUT}}{\partial X_{TestGauge}} \right) u(X_{TestGauge}) \right]^2 + \left[\left(\frac{\partial Y_{UUT}}{\partial E_{Rep}} \right) u(E_{Rep}) \right]^2 + \left[\left(\frac{\partial Y_{UUT}}{\partial E_{Res}} \right) u(E_{Res}) \right]^2} \quad \dots(2)$$

where

$u(X_{TestGauge})$: standard uncertainty for Pressure Test Gauge indication.

$u(E_{Rep})$: standard uncertainty for random variations of Industrial Pressure Gauge indication .

$u(E_{Res})$: standard uncertainty for resolution of Industrial Pressure Gauge indication.

$\frac{\partial Y_{UUT}}{\partial X_{TestGauge}}$, $\frac{\partial Y_{UUT}}{\partial E_{Rep}}$ and $\frac{\partial Y_{UUT}}{\partial E_{Res}}$ are the partial derivatives of Y_{UUT} with respect to

$X_{TestGauge}$,

E_{Rep} , E_{Res} in equation (1). These are the sensitivity coefficients and all are equal to 1.

5.0 TYPE A STANDARD UNCERTAINTY

Maximum standard deviation from Tables 1 and 2 is,

$$s(q_k) = 0.2739 \text{ psi} \quad \dots(3)$$

Hence the Type A standard uncertainty is given by

$$\begin{aligned} u(E_{Rep}) &= \frac{s(q_k)}{\sqrt{n}} \quad \dots(4) \\ &= \frac{0.2739}{\sqrt{5}} \\ &= 0.122 \text{ psi} \end{aligned}$$

6.0 TYPE B STANDARD UNCERTAINTY

6.1 The expanded uncertainty taken from the calibration report of the Pressure Test Gauge is $\pm 0.25\%$ of span with the coverage factor of $k = 2$ at approximately 95 % level of confidence assuming normal distribution.

$$\begin{aligned} a &= \pm (0.25\% \times 100) \text{ psi} \quad \dots(5) \\ &= \pm 0.25 \text{ psi} \end{aligned}$$

EXAMPLE 9: CALIBRATION OF AN INDUSTRIAL PRESSURE GAUGE

Hence the Type B standard uncertainty is given by

$$\begin{aligned}
 u(X_{Test\ Gauge}) &= \frac{a}{2} && \dots(6) \\
 &= \frac{0.25}{2} \\
 &= 0.125\ \text{psi}
 \end{aligned}$$

6.2 The resolution of the Industrial Pressure Gauge indication is half of its divisional interval, which is 0.5 psi and assuming rectangular distribution,

$$\begin{aligned}
 u(E_{Res}) &= \frac{0.5}{\sqrt{12}} && \dots(7) \\
 &= 0.144\ \text{psi}
 \end{aligned}$$

7.0 UNCERTAINTY BUDGET TABLE

Source of Uncertainty	Symbol x_i	Type	Uncertainty value	Probability distribution	Coverage factor	Standard uncertainty $u(x_i)$	Sensitivity coefficient c_i	$ c_i \times u(x_i)$ (psi)	Degrees of freedom
Pressure Test Gauge	$X_{Test\ Gauge}$	B	0.25 psi	Normal	2	0.125 psi	1	0.125	∞
Resolution of Industrial Pressure Gauge	E_{Res}	B	0.5 psi	Rectangular	$\sqrt{12}$	0.144 psi	1	0.144	∞
Repeatability of Industrial Pressure Gauge	E_{Rep}	A	0.122 psi	-	-	0.122 psi	1	0.122	4

8.0 COMBINED STANDARD UNCERTAINTY

The combined standard uncertainty is then given by

$$\begin{aligned}
 u_c(Y_{UUT}) &= \sqrt{u^2(X_{Test\ Gauge}) + u^2(E_{Res}) + u^2(E_{Rep})} && \dots(8) \\
 &= 0.226\ \text{psi}
 \end{aligned}$$

EXAMPLE 9: CALIBRATION OF AN INDUSTRIAL PRESSURE GAUGE

9.0 EFFECTIVE DEGREES OF FREEDOM OF COMBINED STANDARD UNCERTAINTY

The effective degrees of freedom of the combined standard uncertainty is given by:

$$\begin{aligned} \nu_{eff} &= \frac{u_c^4(Y_{UUT})}{\frac{[1 \times u(X_{TestGauge})]^4}{\infty} + \frac{[1 \times u(E_{Res})]^4}{\infty} + \frac{[1 \times u(E_{Rep})]^4}{4}} \quad \dots(9) \\ &= \frac{0.226^4}{\frac{0.125^4}{\infty} + \frac{0.144^4}{\infty} + \frac{0.122^4}{4}} \\ &\approx 47 \end{aligned}$$

10.0 EXPANDED UNCERTAINTY

Since the effective degrees of freedom is greater than 30, $k = 2$ at a confidence level of approximately 95 % is used.

$$\begin{aligned} U &= k u_c(Y_{UUT}) \\ &= 2 \times 0.226 \\ &= 0.5 \text{ psi} \end{aligned}$$

Therefore, the expanded uncertainty is given by

11.0 REPORTING OF RESULTS

Generally, for the Industrial Pressure Gauge indication range of 0 to 100 psi, having an expanded uncertainty of ± 0.5 psi with a coverage factor of $k = 2$ assuming a normal distribution at a level of confidence of approximately 95 %.

EXAMPLE 10: TORQUE TESTER CALIBRATION

1.0 INTRODUCTION

This example demonstrates how the uncertainty of a calibration point for torque tester is evaluated and computed.

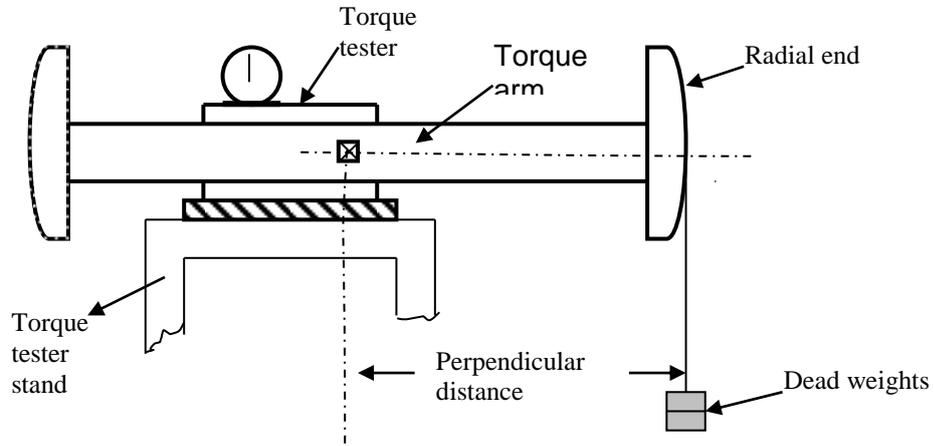


Figure 1 : Set-up of torque tester calibration

The torque tester is calibrated using a torque arm and dead weights as shown in Figure 1. The dead weights are placed at one end of the torque arm, which then produced a desired known torque. The torque arm is designed with a radial end which ensures that the perpendicular distance of the torque arm is maintained, even when it rotates upon loaded.

2.0 MATHEMATICAL MODEL

$$\begin{aligned} \text{Measured Torque} &= \text{Applied Torque} + \text{Error} \\ T_m &= T_a + e \end{aligned} \quad \dots(1)$$

$$\begin{aligned} \text{Applied Torque, } T_a &= \text{Force} \times \text{Perpendicular distance} \\ &= \text{Mass} \times \text{Local Gravity} \times \text{Perpendicular distance} \\ &= M \left(1 - \frac{\rho_a}{\rho_m} \right) g d_{20} (1 + \alpha \theta) \end{aligned} \quad \dots(2)$$

Hence,

$$T_m = M \left(1 - \frac{\rho_a}{\rho_m} \right) g d_{20} (1 + \alpha \theta) + e \quad \dots(3)$$

where

- M : weight of the dead weights applied at the end of the torque arm in kg.
- ρ_a : density of air in kg/m³.
- ρ_m : density of the dead weight material in kg/m³.
- d_{20} : effective length of the torque arm in m at 20 °C as given in its calibration certificate.

EXAMPLE 10: TORQUE TESTER CALIBRATION

- α : coefficient of thermal expansion of the torque arm in $^{\circ}\text{C}^{-1}$.
 θ : deviation of temperature from 20°C reference temperature of the torque arm in $^{\circ}\text{C}$.
 g : local gravity acceleration in m/s^2 .
 e : consists of resolution (δ) and repeatability (T) of the torque tester being calibrated, all in N.m.

Assuming all the above quantities are uncorrelated, equation (3) can be re-written as,

$$T_m = f(M, \rho_a, \rho_m, \alpha, \theta, d_{20}, g, e) \quad \dots(4)$$

Hence the contributory variances are as follows,

$$\begin{aligned} u_c^2(T_m) &= c_M^2 u^2(M) + c_{\rho_a}^2 u^2(\rho_a) + c_{\rho_m}^2 u^2(\rho_m) + c_{\alpha}^2 u^2(\alpha) \\ &\quad + c_{\theta}^2 u^2(\theta) + c_{d_{20}}^2 u^2(d_{20}) + c_g^2 u^2(g) + c_e^2 u^2(e) \end{aligned} \quad \dots(5)$$

with sensitivity coefficients,

$$\begin{aligned} c_M &= \frac{\partial f}{\partial M} \\ &= \left(1 - \frac{\rho_a}{\rho_m}\right) d_{20} (1 + \alpha \theta) g \end{aligned}$$

$$\begin{aligned} c_{\rho_a} &= \frac{\partial f}{\partial \rho_a} \\ &= -\frac{M}{\rho_m} d_{20} (1 + \alpha \theta) g \end{aligned}$$

$$\begin{aligned} c_{\rho_m} &= \frac{\partial f}{\partial \rho_m} \\ &= M \frac{\rho_a}{\rho_m^2} d_{20} (1 + \alpha \theta) g \end{aligned}$$

$$\begin{aligned} c_g &= \frac{\partial f}{\partial g} \\ &= M \left(1 - \frac{\rho_a}{\rho_m}\right) d_{20} (1 + \alpha \theta) \end{aligned}$$

EXAMPLE 10: TORQUE TESTER CALIBRATION

$$\begin{aligned}c_{d_{20}} &= \frac{\partial f}{\partial d_{20}} \\ &= M \left(1 - \frac{\rho_a}{\rho_m} \right) (1 + \alpha \theta) \text{ g}\end{aligned}$$

$$\begin{aligned}c_{\alpha} &= \frac{\partial f}{\partial \alpha} \\ &= \theta d_{20} M \left(1 - \frac{\rho_a}{\rho_m} \right) \text{ g}\end{aligned}$$

$$\begin{aligned}c_{\theta} &= \frac{\partial f}{\partial \theta} \\ &= \alpha d_{20} M \left(1 - \frac{\rho_a}{\rho_m} \right) \text{ g}\end{aligned}$$

$$\begin{aligned}c_e &= \frac{\partial f}{\partial e} \\ &= 1\end{aligned}$$

In this example, a calibration point at 12 Nm is chosen on a torque tester which measuring range is 0 to 12 Nm.

3.0 TYPE B UNCERTAINTY EVALUATION

Calculated weight to be applied is 4.910 kg to produce the required known torque. The uncertainty computed using a coverage factor of $k = 2$ is 0.025 kg.

The standard uncertainty is then

$$\begin{aligned}u(M) &= \frac{0.025 \text{ kg}}{2} \\ &= 12.5 \times 10^{-3} \text{ kg}\end{aligned}$$

The Density of dead weights, ρ_m is given by the manufacturer to be 8000 kg/m^3 with a rectangular distribution with bounds $\pm 100 \text{ kg/m}^3$. The standard uncertainty is then

$$\begin{aligned}u(\rho_m) &= \frac{100 \text{ kg/m}^3}{\sqrt{3}} \\ &= 57.74 \text{ kg/m}^3\end{aligned}$$

EXAMPLE 10: TORQUE TESTER CALIBRATION

The local gravity acceleration, g is calibrated to be $9.78065 \pm 0.00005 \text{ m/s}^2$ using a coverage factor of $k = 2$. The standard uncertainty is then

$$\begin{aligned}u(g) &= \frac{0.00005 \text{ m/s}^2}{2} \\ &= 2.5 \times 10^{-5} \text{ m/s}^2\end{aligned}$$

The length of torque arm, d_{20} is calibrated to be $0.25000 \pm 0.00002 \text{ m}$ using a coverage factor of $k = 2$. The standard uncertainty is then

$$\begin{aligned}u(d_{20}) &= \frac{0.00002 \text{ m}}{2} \\ &= 10 \times 10^{-6} \text{ m}\end{aligned}$$

Coefficient of thermal expansion, α of the torque arm is given by the manufacturer to be $1 \times 10^{-7} \text{ m / }^\circ\text{C}$. Since the value of α is very small, its standard uncertainty contribution to the combined uncertainty is neglected, hence

$$u(\alpha) = 0$$

The temperature of the environment is controlled and monitored to be $20 \text{ }^\circ\text{C}$, hence $\theta = 0$. The uncertainty represented by a rectangular distribution with bounds $\pm 1 \text{ }^\circ\text{C}$. The standard uncertainty is then

$$\begin{aligned}u(\theta) &= \frac{1 \text{ }^\circ\text{C}}{\sqrt{3}} \\ &= 0.5774 \text{ }^\circ\text{C}\end{aligned}$$

Within the laboratory controlled ambient condition, the air density of 1.2 kg/m^3 would have an uncertainty represented by a rectangular distribution with bounds $\pm 0.06 \text{ kg/m}^3$. The standard uncertainty is then,

$$\begin{aligned}u(\rho_a) &= \frac{0.06 \text{ kg/m}^3}{\sqrt{3}} \\ &= 0.03464 \text{ kg/m}^3\end{aligned}$$

Standard uncertainty of indication of torque tester display is,

$$\begin{aligned}u(\delta) &= \frac{\delta}{2\sqrt{3}} \\ &= \frac{0.01}{2\sqrt{3}} \\ &= 2.88 \times 10^{-3} \text{ Nm}\end{aligned}$$

where δ = resolution of torque tester display

EXAMPLE 10: TORQUE TESTER CALIBRATION

4.0 TYPE A UNCERTAINTY EVALUATION

10 independent repeated observations were taken. The arithmetic mean of these readings is computed to be 11.85 Nm with a standard uncertainty of

$$\begin{aligned}u(T) &= s\left(\overline{T}\right) \\ &= \frac{\text{Standard deviation}}{\sqrt{n}} \\ &= \frac{0.05 \text{ Nm}}{\sqrt{10}} \\ &= 1.58 \times 10^{-2} \text{ Nm}\end{aligned}$$

where n = number of independent repeated observations.

$$u^2(e) = u^2(T) + u^2(\delta) \quad \dots(6)$$

5.0 SENSITIVITY COEFFICIENTS

The sensitivity coefficients in equation (5) are computed as,

$$\begin{aligned}\frac{\partial f}{\partial \alpha} &= 0 \times 0.25 \times 4.91 \left(1 - \frac{1.2}{8000}\right) 9.78065 \\ &= 0\end{aligned}$$

$$\begin{aligned}\frac{\partial f}{\partial \theta} &= 1 \times 10^{-7} \times 0.25 \times 4.91 \left(1 - \frac{1.2}{8000}\right) 9.78065 \\ &= 1.20 \times 10^{-6}\end{aligned}$$

$$\begin{aligned}\frac{\partial f}{\partial M} &= \left(1 - \frac{1.2}{8000}\right) 0.25 (1 + 1 \times 10^{-7} \times 0) 9.78065 \\ &= 2.445\end{aligned}$$

$$\begin{aligned}\frac{\partial f}{\partial \rho_a} &= -\frac{4.91}{8000} [0.25 (1 + 1 \times 10^{-7} \times 0)] 9.78065 \\ &= -0.002\end{aligned}$$

$$\begin{aligned}\frac{\partial f}{\partial \rho_m} &= 4.91 \left(\frac{1.2}{8000^2}\right) [0.25 (1 + 1 \times 10^{-7} \times 0)] 9.78065 \\ &= 2.25 \times 10^{-7}\end{aligned}$$

EXAMPLE 10: TORQUE TESTER CALIBRATION

$$\begin{aligned}\frac{\partial f}{\partial g} &= 4.91 \left(1 - \frac{1.2}{8000} \right) 0.25 (1 + 1 \times 10^{-7} \times 0) \\ &= 1.227\end{aligned}$$

$$\begin{aligned}\frac{\partial f}{\partial d_{20}} &= 4.91 \left(1 - \frac{1.2}{8000} \right) (1 + 1 \times 10^{-7} \times 0) 9.78065 \\ &= 48.016\end{aligned}$$

6.0 UNCERTAINTY BUDGET TABLE

Source of Uncertainty	Symbol x_i	Type	Uncertainty value	Probability distribution	Coverage factor	Standard uncertainty $u(x_i)$	Sensitivity coefficient c_i	$ c_i \times u(x_i)$ Nm	Degrees of freedom ν
Weights	M	B	25×10^{-3} kg	Normal	2	12.5×10^{-3} kg	2.445	3.06×10^{-2}	∞
Density of weights	ρ_m	B	100 kg/m ³	Rectangular	$\sqrt{3}$	57.74 kg/m ³	2.25×10^{-7}	2.25×10^{-5}	∞
Gravity acceleration	g	B	5×10^{-5} m/s ²	Normal	2	2.5×10^{-5} m/s ²	1.227	3.07×10^{-5}	∞
Length of arm	d_{20}	B	2×10^{-5} m	Normal	2	10×10^{-6} m	48.016	4.80×10^{-4}	∞
Coefficient of Thermal Expansion	α	B	-	-	-	0	0	0	-
Temperature	θ	B	1 °C	Rectangular	$\sqrt{3}$	5.774×10^{-1} °C	1.2×10^{-6}	6.92×10^{-7}	∞
Air density	ρ_a	B	6×10^{-2} kg/m ³	Rectangular	$\sqrt{3}$	3.464×10^{-2} kg/m ³	-2×10^{-3}	-6.93×10^{-5}	∞
Resolution	δ	B	5×10^{-3} Nm	Rectangular	$\sqrt{3}$	2.88×10^{-3} Nm	1	2.88×10^{-3}	∞
Repeatability	T	A	1.58×10^{-2} Nm	-	-	1.58×10^{-2} Nm	1	1.58×10^{-2}	9

7.0 COMBINED STANDARD UNCERTAINTY

Collection of the individual terms discussed previously and substitute them and equation (6) into equation (5) yields for the variance and the combined standard uncertainty, $u_c(T_m)$.

$$\begin{aligned}u_c^2(T_m) &= 2.445^2 (12.5 \times 10^{-3})^2 + (-0.002)^2 0.03464^2 + (2.25 \times 10^{-7})^2 \times 57.74^2 \\ &\quad + 48.016^2 (10 \times 10^{-6})^2 + 0 \times 0 + (1.2 \times 10^{-6})^2 0.5774^2 + 1.227^2 (2.5 \times 10^{-5})^2 \\ &\quad + (1.58 \times 10^{-2})^2 + (2.88 \times 10^{-3})^2 \\ &= 11.92 \times 10^{-4} \\ u_c(T_m) &= \sqrt{11.92 \times 10^{-4}} \\ &= 0.035 \text{ Nm}\end{aligned}$$

EXAMPLE 10: TORQUE TESTER CALIBRATION

8.0 EFFECTIVE DEGREES OF FREEDOM

Suppose that one is required to obtain an expanded uncertainty $U = k u_c(T_m)$ that provides a level of confidence of approximately 95 %. The procedure is to first compute the effective degrees of freedom, $\nu_{eff}(T_m)$.

To obtain the effective degrees of freedom, ν_i the degrees of freedom for each standard uncertainty component is required. For a component obtained from Type A evaluation, ν_i is obtained from the number of independent repeated observations upon which the corresponding input estimate is based and the number of independent quantities determined from those observations.

For a component obtained from a Type B evaluation, ν_i is obtained from the judged reliability of the value of that component, which is often the case in practise, $\nu_i \rightarrow \infty$, such that $\nu(M)$, $\nu(\rho_m)$, $\nu(\alpha)$, $\nu(\theta)$, $\nu(\rho_a)$, $\nu(\delta)$, $\nu(d_{20})$ and $\nu(g)$ are treated as $\rightarrow \infty$. Hence,

$$\begin{aligned}\nu_{eff}(T_m) &= \frac{[u_c(T_m)]^4}{\frac{u^4(T)}{\nu(T)}} \\ &= \frac{0.035^4}{\frac{(1.58 \times 10^{-2})^4}{9}} \\ &= 216\end{aligned}$$

9.0 EXPANDED UNCERTAINTY

For $\nu_{eff}(T_m) = 216$, $k = 2$ can be used at approximately 95 % level of confidence.

$$\begin{aligned}\text{Expanded Uncertainty, } U &= k u_c(T_m) \\ &= 2 \times 0.035 \\ &= 0.07 \text{ Nm}\end{aligned}$$

10.0 REPORTING OF RESULTS

The measured torque of 11.85 Nm having an expanded uncertainty of ± 0.07 Nm with a coverage factor of $k = 2$, assuming a normal distribution at a level of confidence of approximately 95 %.

EXAMPLE 11: CALIBRATION OF WEIGHING MACHINE

1.0 INTRODUCTION

An electronic weighing machine with a scale of range, 0 to 60 kg and resolutions of 0.01 kg was calibrated by direct comparison method, using a set of OIML class M₂ standard weight. Prior to calibration, the weighing machine was adjusted at full load using a standard weight according to the manufacturer procedure. The repeatability of the weighing machine was determined at near zero, half-full load, full load and the pool standard deviation was obtained. Normally the scale reading of the weighing machine would be calibrated by placing standard weights on the receptor in increasing increments of about 10% of the full capacity. In this example the measurement uncertainty for the weighing machine is calculated at the full-applied load. During the calibration, when the 60 kg standard weight applied on the weighing machine it indicated 59.99 kg.

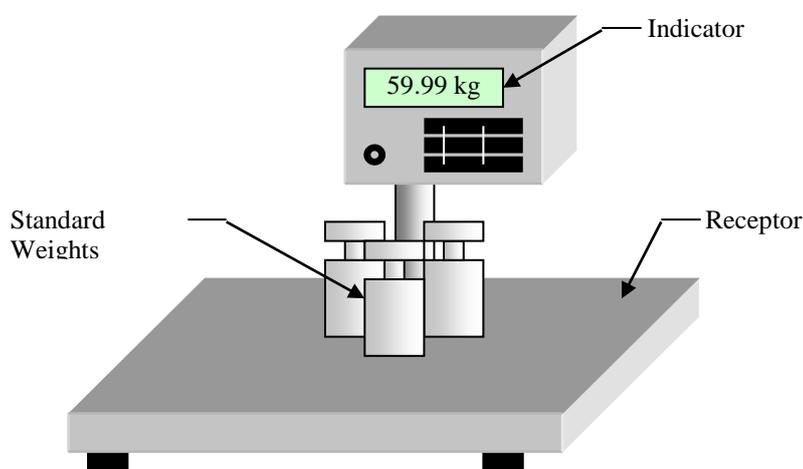


Figure 1: Weighing Machine being calibrated with 60 kg (built up by three 20 kg) standard weights

2.0 MATHEMATICAL MODEL

The mathematical model for direct comparison method can be expressed as:

$$S = M + E \quad \dots(1)$$

where

S : Scale reading (the reading of the unit under test).

M : Mass of the standard weight used to calibrate the electronic weighing machine.

E : Error of the scale reading.

3.0 LAW OF PROPAGATION OF UNCERTAINTY

Where all input quantities are considered uncorrelated or called independent, the combined standard uncertainty $u_c(S)$ can be expressed by law of propagation of the scale reading (given in equation (1)) as follows:

EXAMPLE 11: CALIBRATION OF WEIGHING MACHINE

$$u_c^2(S) = \left(\frac{\partial S}{\partial M} \right)^2 u^2(M) + \left(\frac{\partial S}{\partial E} \right)^2 u^2(E)$$

where each corresponding sensitivity coefficient is as follows

$$\begin{aligned} c_M &= \frac{\partial S}{\partial M} \\ &= 1 \end{aligned}$$

$$\begin{aligned} c_E &= \frac{\partial S}{\partial E} \\ &= 1 \end{aligned}$$

$u(M)$: standard uncertainty of the standard weight.

$u(E)$: standard uncertainty due to the error of the scale reading.

and

$$u(M) = \sqrt{u^2(M_c) + u^2(M_d)}$$

where

$u(M_c)$: standard uncertainty of the standard weight taken from its calibration report.

$u(M_d)$: standard uncertainty of the standard weight drift within its calibration intervals.

and

$$u(E) = \sqrt{u^2(E_{res}) + u^2(E_b) + u^2(E_{rep})}$$

where

$u(E_{res})$: standard uncertainty of the error of the scale reading due to its resolution.

$u(E_b)$: standard uncertainty of the error of the scale reading due to air buoyancy effect.

$u(E_{rep})$: standard uncertainty of the error of the scale reading due to its repeatability.

Therefore,

$$u_c^2(S) = c_M^2 \times u^2(M_c) + c_M^2 \times u^2(M_d) + c_E^2 \times u^2(E_{res}) + c_M^2 \times u^2(E_b) + c_E^2 \times u^2(E_{rep}) \quad \dots(2)$$

4.0 EVALUATION OF UNCERTAINTY COMPONENTS

Type B

- 4.1 u_{M_c} , a set of standard weights was used to calibrate the weighing machine from 0 to 60 kg, the largest uncertainty would be at the largest built up standard weight of 60 kg with the uncertainty of ± 1.8 g, at the given level of confidence of 95 %. Assuming a normal distribution, the degrees of freedom is ∞ .

EXAMPLE 11: CALIBRATION OF WEIGHING MACHINE

Hence, the standard uncertainty

$$\begin{aligned}u_{M_c} &= \pm \left(\frac{1.8}{2} \right) \\ &= \pm 0.9 \text{ g}\end{aligned}$$

- 4.2 u_{M_d} , the estimated limits of the standard weights drift within the calibration interval is ± 0.3 g. It is assumed to be a rectangular distribution, hence the degrees of freedom is ∞ . Thus, the standard uncertainty

$$\begin{aligned}u_{M_d} &= \pm \left(\frac{0.3}{\sqrt{3}} \right) \\ &= \pm 0.173 \text{ g}\end{aligned}$$

- 4.3 $u_{E_{res}}$, resolutions of the unit under test is 0.01 kg considered as a rectangular distribution, hence the degrees of freedom is ∞ . Therefore, the standard uncertainty,

$$\begin{aligned}u_{E_{res}} &= \pm \left(\frac{\frac{10}{2}}{\sqrt{3}} \right) \text{ g} \\ &= \pm \left(\frac{5}{\sqrt{3}} \right) \text{ g} \\ &= \pm 2.9 \text{ g}\end{aligned}$$

- 4.4 u_{E_b} , the calibration was carry out in the controlled environment laboratory, the standard uncertainty of the weighing machine reading due to air bouyancy effect is estimated to be ± 0.55 g, assuming a rectangular distribution with degrees of freedom equals ∞ . The standard uncertainty

$$\begin{aligned}u_{E_b} &= \pm \left(\frac{0.55}{\sqrt{3}} \right) \text{ g} \\ &= \pm 0.318 \text{ g}\end{aligned}$$

Type A

- 4.5 $u_{E_{rep}}$, standard uncertainty during the calibration were obtained from three intervals of the scale. At near zero, the standard deviation, $\sigma_0 = \pm 0.01$ kg with the degrees of freedom, $\nu_0 = 9$. At half full load, the standard deviation, $\sigma_h = \pm 0.01$ kg with the degrees of freedom, $\nu_h = 9$. At full load, the standard deviation, $\sigma_f = \pm 0.01$ kg with the degrees of freedom, $\nu_f = 9$.

The pooled standard deviation ,

$$\begin{aligned}\sigma_{E_{rep}} &= \pm \sqrt{\frac{\nu_0 \times \sigma_0^2 + \nu_h \times \sigma_h^2 + \nu_f \times \sigma_f^2}{\nu_0 + \nu_h + \nu_f}} \\ &= \pm \sqrt{\frac{9 \times 0.01^2 + 9 \times 0.01^2 + 9 \times 0.01^2}{9 + 9 + 9}} \\ &= \pm 10 \text{ g}\end{aligned}$$

EXAMPLE 11: CALIBRATION OF WEIGHING MACHINE

with the degrees of freedom $\nu_{E_{rep}} = 27$. Since the scale reading during calibration was measured one time at each calibrated point, hence, the standard uncertainty

$$u_{E_{rep}} = \pm \left(\frac{\sigma_{E_{rep}}}{\sqrt{1}} \right)$$

$$= \pm 10 \text{ g}$$

5.0 UNCERTAINTY BUDGET TABLE

Source of Uncertainty	Symbol x_i	Type	Uncertainty value	Probability distribution	Coverage factor	Standard uncertainty $u(x_i)$	Sensitivity coefficient c_i	$ c_i \times u(x_i)$	Degrees of freedom ν
Standard weight from calibration	M_c	B	1.8 g	Normal	2	0.9 g	1	0.9 g	∞
Standard weight drift	M_d	B	0.3 g	Rectangular	$\sqrt{3}$	0.173 g	1	0.173 g	∞
Resolution of scale reading	E_{res}	B	5 g	Rectangular	$\sqrt{3}$	2.9 g	1	2.9 g	∞
Air bouyancy effect on scale reading	E_b	B	0.55 g	Rectangular	$\sqrt{3}$	0.318 g	1	0.318 g	∞
Repeatability of scale reading	E_{rep}	A	10 g	-	-	10 g	1	10 g	27

6.0 COMBINED STANDARD UNCERTAINTY

The Combined Standard Uncertainty $u_c(S)$ using equation (2)

$$u_c^2(S) = c_M^2 \times u^2(M_c) + c_M^2 \times u^2(M_d) + c_E^2 \times u^2(E_{res}) + c_M^2 \times u^2(E_b) + c_E^2 \times u^2(E_{rep})$$

$$= [c_M \times u(M_c)]^2 + [c_M \times u(M_d)]^2 + [c_E \times u(E_{res})]^2 + [c_E \times u(E_b)]^2 + [c_E \times u(E_{rep})]^2$$

$$= 0.9^2 + 0.173^2 + 2.9^2 + 0.318^2 + 10^2$$

$$u_c(S) = \pm 10.454 \text{ g}$$

7.0 EFFECTIVE DEGREES OF FREEDOM

Using Welch-Satterthwaite formula to calculate the effective degrees of freedom for combined standard uncertainty

EXAMPLE 11: CALIBRATION OF WEIGHING MACHINE

$$\begin{aligned}v_{eff} &= \frac{u_c^4(S)}{\frac{[c_M \times u(M_c)]^4}{v_{M_c}} + \frac{[c_M \times u(M_d)]^4}{v_{M_d}} + \frac{[c_E \times u(E_{Res})]^4}{v_{E_{Res}}} + \frac{[c_E \times u(E_b)]^4}{v_{E_b}} + \frac{[c_E \times u(E_{rep})]^4}{v_{E_{rep}}}} \\ &= \frac{10.452^4}{\frac{0.9^4}{\infty} + \frac{0.173^4}{\infty} + \frac{2.9^4}{\infty} + \frac{0.006^4}{\infty} + \frac{10^4}{27}} \\ &\approx 32\end{aligned}$$

where

- v_{eff} : effective degrees of freedom of the combined standard uncertainty.
- v_{M_c} : degrees of freedom of the standard uncertainty of standard weight.
- v_{M_d} : degrees of freedom of the standard uncertainty due to standard weight drift.
- $v_{E_{res}}$: degrees of freedom of the standard uncertainty due to scale reading resolution.
- v_{E_b} : degrees of freedom of the standard uncertainty due to air buoyancy on scale reading.
- $v_{E_{rep}}$: degrees of freedom of the standard uncertainty due to scale reading repeatability.

8.0 EXPANDED UNCERTAINTY

For $v_{eff} = 32$, $k = 2$ can be used at approximately 95 % level of confidence. Therefore, the expanded uncertainty $U(S)$ is then calculated as follows:

$$\begin{aligned}U(S) &= \pm k u_c(S) \\ &= \pm 2 \times 10.451 \text{ g} \\ &= \pm 20.902 \text{ g} \\ &\approx \pm 0.02 \text{ kg}\end{aligned}$$

9.0 REPORTING OF RESULTS

During the calibration, when the 60 kg standard weight applied on the weighing machine it indicated 59.99 kg.

The weighing machine indicated value of 59.99 kg having an expanded uncertainty of ± 0.02 kg with a coverage factor of $k = 2$, assuming a normal distribution at a level of confidence of approximately 95 %.

EXAMPLE 12: COORDINATE MEASURING MACHINE MEASUREMENT

1.0 INTRODUCTION

Using a coordinate measuring machine (CMM) to measure the length of a cylindrical rod from the centre of one end face to the other.

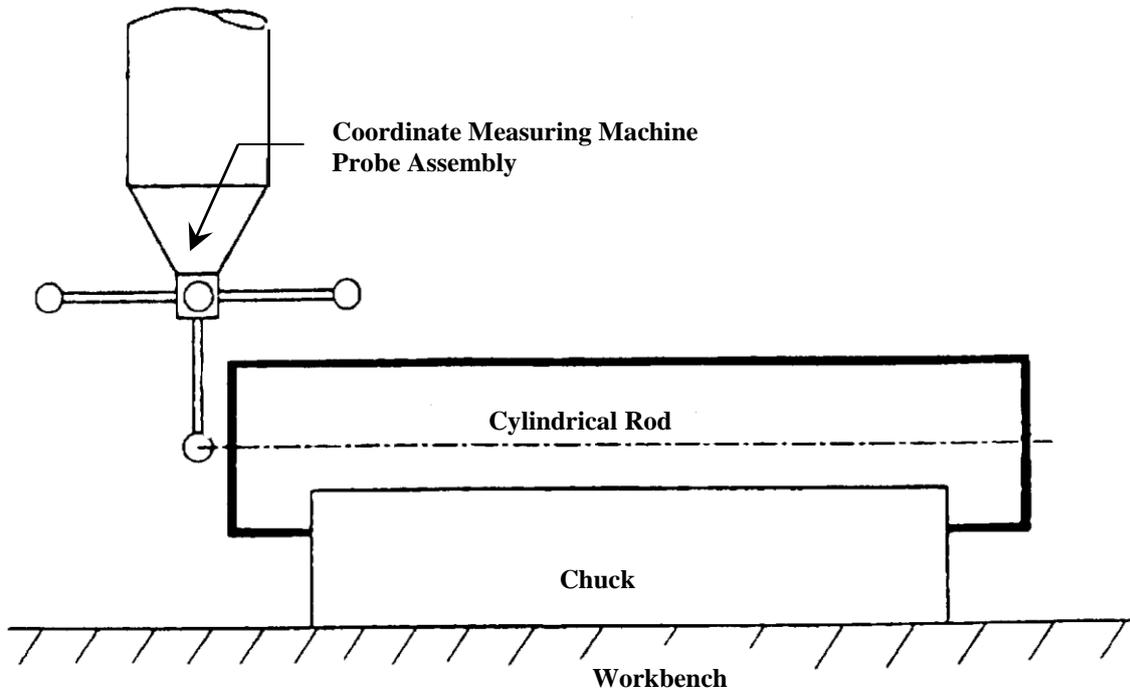


Figure 1: Measurement set-up

2.0 MATHEMATICAL MODEL

$$L_r = L_m (1 - \alpha_m \Delta T_m + \alpha_r \Delta T_r) \quad \dots(9)$$

where

L_r : length of cylindrical rod at 20 °C.

L_m : length as measured by CMM.

α_r : coefficient of thermal expansion of cylindrical rod.

α_m : coefficient of thermal expansion of CMM scale.

ΔT_r : temperature deviation of the cylindrical rod from 20 °C during measurement process.

ΔT_m : temperature deviation of CMM scale from 20 °C during measurement process.

Contributory variances :

$$u_c^2(L_r) = \left(\frac{\partial L_r}{\partial L_m} \right)^2 u^2(L_m) + \left(\frac{\partial L_r}{\partial \alpha_m} \right)^2 u^2(\alpha_m) + \left(\frac{\partial L_r}{\partial \Delta T_m} \right)^2 u^2(\Delta T_m) + \left(\frac{\partial L_r}{\partial \alpha_r} \right)^2 u^2(\alpha_r) + \left(\frac{\partial L_r}{\partial \Delta T_r} \right)^2 u^2(\Delta T_r) \quad \dots(2)$$

EXAMPLE 12: COORDINATE MEASURING MACHINE MEASUREMENT

Sensitivity Coefficients are as follows :-

$$\frac{\partial L_r}{\partial L_m} = 1 - \alpha_m \Delta T_m + \alpha_r \Delta T_r$$

$$\frac{\partial L_r}{\partial \alpha_m} = -L_m \Delta T_m$$

$$\frac{\partial L_r}{\partial \Delta T_m} = -L_m \alpha_m$$

$$\frac{\partial L_r}{\partial \alpha_r} = L_m \Delta T_r$$

$$\frac{\partial L_r}{\partial \Delta T_r} = L_m \alpha_r$$

Therefore,

$$u_c^2(L_r) = (1 - \alpha_m \Delta T_m + \alpha_r \Delta T_r)^2 u^2(L_m) + (-L_m \Delta T_m)^2 u^2(\alpha_m) \\ + (-L_m \alpha_m)^2 u^2(\Delta T_m) + (L_m \Delta T_r)^2 u^2(\alpha_r) + (L_m \alpha_r)^2 u^2(\Delta T_r)$$

3.0 STANDARD UNCERTAINTY EVALUATION

3.1 Uncertainty of measured length, $u(L_m)$

From the ten repeated measurements taken, the mean measured result was 300.0008 mm with a standard deviation of 0.0020 mm. Therefore, the standard uncertainty, $u(L_{m1})$, equals the standard deviation of the mean of the ten measurements, 0.0006 mm, with degrees of freedom, $\nu(L_{m1}) = 10 - 1 = 9$.

From the performance verification report on the CMM, the measurement uncertainty of the machine at a confidence level of approximately 95 % is $\pm (1.2 + 3l) \mu\text{m}$, where l is in metre. Therefore, the standard uncertainty, $u(L_{m2}) = (1.2 + 3 \times 0.3) / 2 = 1.05 \mu\text{m}$ or 0.00105 mm with degrees of freedom, $\nu(L_{m2}) = \infty$, normal distribution is assumed.

Therefore,

$$u^2(L_m) = u^2(L_{m1}) + u^2(L_{m2}) \\ = 0.0006^2 + 0.00105^2 \\ u(L_m) = 0.001209 \text{ mm}$$

EXAMPLE 12: COORDINATE MEASURING MACHINE MEASUREMENT

with degrees of freedom, $\nu(L_m)$

$$\begin{aligned}\nu(L_m) &= \frac{u^4(L_m)}{\frac{u^4(L_{m1})}{\nu(L_{m1})} + \frac{u^4(L_{m2})}{\nu(L_{m2})}} \\ &= \frac{0.001209^4}{\frac{0.0006^4}{9} + \frac{0.00105^4}{\infty}} \\ &= 148\end{aligned}$$

3.2 Uncertainty of coefficient of thermal expansion of CMM scale, $u(\alpha_m)$

The coefficient of thermal expansion of CMM scale is given as $\alpha_m = 5 \times 10^{-6} / ^\circ\text{C}$ with an uncertainty represented by a rectangular distribution with boundaries $\pm 2 \times 10^{-6} / ^\circ\text{C}$. Therefore, the standard uncertainty,

$$\begin{aligned}u(\alpha_m) &= \frac{(2 \times 10^{-6} / ^\circ\text{C})}{\sqrt{3}} \\ &= 1.2 \times 10^{-6} / ^\circ\text{C} \quad \text{with degrees of freedom } \nu(\alpha_m) = \infty\end{aligned}$$

3.3 Uncertainty of temperature deviation of CMM scale from 20 °C, $u(\Delta T_m)$.

The temperature of CMM scale was measured as 19.9 °C with an uncertainty represented by a rectangular distribution with boundaries ± 0.5 °C. The standard uncertainty,

$$\begin{aligned}u(\Delta T_m) &= \frac{0.5}{\sqrt{3}} \\ &= 0.289 \text{ } ^\circ\text{C} \quad \text{with degrees of freedom } \nu(\Delta T_m) = \infty\end{aligned}$$

4.0 SENSITIVITY COEFFICIENTS

$$\begin{aligned}\frac{\partial L_r}{\partial L_m} &= 1 - \alpha_m \Delta T_m + \alpha_r \Delta T_r \\ &= 1 + 5 \times 10^{-6} \times 0.1 - 11.5 \times 10^{-6} \times 0.1 = 0.999999\end{aligned}$$

$$\begin{aligned}\frac{\partial L_r}{\partial \alpha_m} &= -L_m \Delta T_m \\ &= 300.0008 \times 0.1 = 30.00008 \text{ mm } ^\circ\text{C}\end{aligned}$$

EXAMPLE 12: COORDINATE MEASURING MACHINE MEASUREMENT

$$\begin{aligned}\frac{\partial L_r}{\partial \Delta T_m} &= -L_m \alpha_m \\ &= -300.0008 \times 5 \times 10^{-6} = -0.0015 \text{ mm}/^\circ\text{C}\end{aligned}$$

$$\begin{aligned}\frac{\partial L_r}{\partial \alpha_r} &= L_m \Delta T_r \\ &= -300.0008 \times 0.1 = -30.00008 \text{ mm}^\circ\text{C}\end{aligned}$$

$$\begin{aligned}\frac{\partial L_r}{\partial \Delta T_r} &= L_m \alpha_r \\ &= 300.0008 \times 11.5 \times 10^{-6} = 0.00345 \text{ mm}/^\circ\text{C}\end{aligned}$$

5.0 UNCERTAINTY BUDGET TABLE

Source of Uncertainty	Symbol x_i	Type	Uncertainty value	Probability distribution	Coverage factor	Standard uncertainty $u(x_i)$	Sensitivity coefficient c_i	$ c_i \times u(x_i)$ mm	Degrees of freedom ν
Repeatability of reading	$u(L_{m1})$	A	-	-	-	0.6 μm	0.999 999	0.0006	9
CMM specification	$u(L_{m2})$	B	$\pm (1.2 + 3 l)$ μm	Normal	1.96	1.05 μm	0.999 999	0.0010	∞
Coefficient of thermal expansion of CMM scale	$u(\alpha_m)$	B	$\pm 2 \times 10^{-6} / ^\circ\text{C}$	Rectangular	$\sqrt{3}$	$\pm 1.2 \times 10^{-6} / ^\circ\text{C}$	30.000 08 mm $^\circ\text{C}$	3.6×10^{-5}	∞
Temperature deviation of CMM scale	$u(\Delta T_m)$	B	$\pm 0.5 ^\circ\text{C}$	Rectangular	$\sqrt{3}$	0.289 $^\circ\text{C}$	- 0.0015 mm / $^\circ\text{C}$	0.0004	∞
Coefficient of thermal expansion of rod	$u(\alpha_r)$	B	$\pm 2 \times 10^{-6} / ^\circ\text{C}$	Rectangular	$\sqrt{3}$	$\pm 1.2 \times 10^{-6} / ^\circ\text{C}$	- 30.000 08 mm $^\circ\text{C}$	3.6×10^{-5}	∞
Temperature deviation of rod	$u(\Delta T_r)$	B	$\pm 0.5 ^\circ\text{C}$	Rectangular	$\sqrt{3}$	0.289 $^\circ\text{C}$	0.00345 mm / $^\circ\text{C}$	0.0010	∞

6.0 COMBINED STANDARD UNCERTAINTY, $u_c(L_r)$

$$\begin{aligned}u_c^2(L_r) &= (1 - \alpha_m \Delta T_m + \alpha_r \Delta T_r)^2 u^2(L_m) + (-L_m \Delta T_m)^2 u^2(\alpha_m) + (-L_m \alpha_m)^2 u^2(\Delta T_m) \\ &\quad + (L_m \Delta T_r)^2 u^2(\alpha_r) + (L_m \alpha_r)^2 u^2(\Delta T_r) \\ &= 0.999999^2 \times 0.001209^2 + 30.00008^2 \times (1.2 \times 10^{-6})^2 + 0.0015^2 \times 0.289^2 \\ &\quad + 30.00008^2 \times (1.2 \times 10^{-6})^2 + 0.00345^2 \times 0.289^2 \\ &= 0.0000026 \text{ mm}^2 \\ u_c(L_r) &= 0.00161 \text{ mm}\end{aligned}$$

EXAMPLE 12: COORDINATE MEASURING MACHINE MEASUREMENT

With effective degrees of freedom,

$$\begin{aligned} \nu_{eff} &= \frac{u_c^4(L_r)}{\frac{[(1 - \alpha_m \Delta T_m + \alpha_r \Delta T_r)u(L_m)]^4}{\nu(L_m)} + \frac{[(-L_m \Delta T_m)u(\alpha_m)]^4}{\nu(\alpha_m)} + \frac{[(-L_m \alpha_m)u(\Delta T_m)]^4}{\nu(\Delta T_m)} + \frac{[(L_m \Delta T_r)u(\alpha_r)]^4}{\nu(\alpha_r)} + \frac{[(L_m \alpha_r)u(\Delta T_r)]^4}{\nu(\Delta T_r)}} \\ &= \frac{0.00161^4}{\frac{[(1 + 5 \times 10^{-6} \times 0.1 - 11.5 \times 10^{-6} \times 0.1) \times 0.001209]^4}{148} + 0 + 0 + 0 + 0} \\ &= 465 \end{aligned}$$

7.0 EXPANDED UNCERTAINTY

For $\nu_{eff}(L_r) = 465$, $k = 2$ can be used at an approximately 95 % level of confidence level.

Therefore,

$$\begin{aligned} U &= k u_c \\ &= 2 \times 0.00161 \\ &= 0.0032 \text{ mm} \end{aligned}$$

8.0 MEAN MEASURED RESULT

$$\begin{aligned} L_r &= L_m (1 - \alpha_m \Delta T_m + \alpha_r \Delta T_r) \\ &= 300.0008(1 + 5 \times 10^{-6} \times 0.1 - 11.5 \times 10^{-6} \times 0.1) \\ &= 300.0006 \text{ mm} \end{aligned}$$

9.0 REPORTING OF RESULTS

The measured length of the cylindrical rod was found to be 300.0006 mm. The associated expanded uncertainty of measurement is ± 0.0032 mm, estimated at a level of confidence of approximately 95 % with coverage factor $k = 2$.

EXAMPLE 13: MICROMETER CALIBRATION

1.0 INTRODUCTION

The calibrated instrument is a digital micrometer of 0-25 mm range with resolution of 0.001 mm.

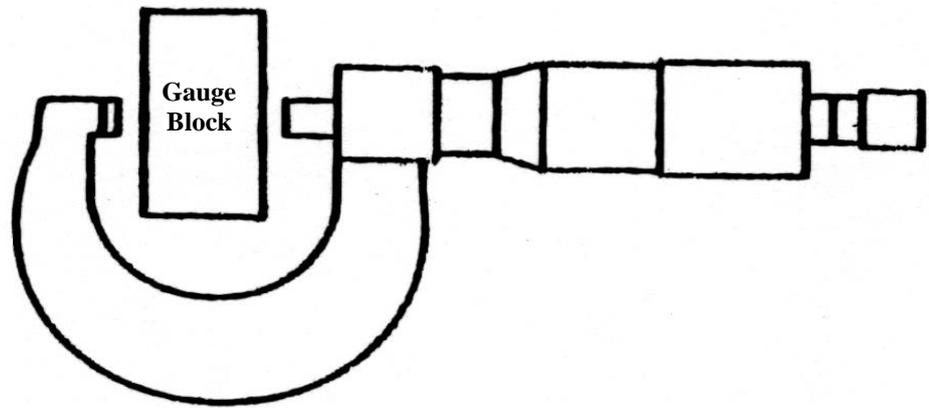


Figure 1: Measurement set-up

The error of measurement is obtained by comparison against gauge blocks of the following sizes:

3.1, 6.5, 9.7, 12.5, 15.8, 19, 21.9, 25 mm

2.0 MATHEMATICAL MODEL

$$E_{20} = IR_{20} - G_{20} \quad \dots(1)$$

where

E_{20} : error of measurement obtained at 20 °C.

IR_{20} : instrument reading at 20 °C.

G_{20} : gauge block value given in the calibration report at 20 °C.

With consideration of the temperature effect on the instrument under calibration, the above expression can be re-written as:

$$E_{20} = IR_t (1 - \alpha_{IR} \Delta t_{IR}) - G_{20} \quad \dots(2)$$

where

IR_t : instrument reading at temperature t .

α_{IR} : coefficient of thermal expansion of the instrument.

Δt_{IR} : temperature deviation of instrument from 20 °C, ie $(t - 20)$.

EXAMPLE 13: MICROMETER CALIBRATION

Contributory variances:

$$u_c^2(E_{20}) = \left(\frac{\partial E_{20}}{\partial IR_t}\right)^2 u^2(IR_t) + \left(\frac{\partial E_{20}}{\partial \alpha_{IR}}\right)^2 u^2(\alpha_{IR}) + \left(\frac{\partial E_{20}}{\partial \Delta t_{IR}}\right)^2 u^2(\Delta t_{IR}) + \left(\frac{\partial E_{20}}{\partial G_{20}}\right)^2 u^2(G_{20}) \quad \dots(3)$$

Sensitivity coefficients:

$$\begin{aligned} \left(\frac{\partial E_{20}}{\partial IR_t}\right) &= 1 - \alpha_{IR} \Delta t_{IR} \\ \left(\frac{\partial E_{20}}{\partial \alpha_{IR}}\right) &= -IR_t \Delta t_{IR} \\ \left(\frac{\partial E_{20}}{\partial \Delta t_{IR}}\right) &= -IR_t \alpha_{IR} \\ \left(\frac{\partial E_{20}}{\partial G_{20}}\right) &= -1 \end{aligned}$$

Therefore,

$$u_c^2(E_{20}) = (1 - \alpha_{IR} \Delta t_{IR})^2 u^2(IR_t) + (IR_t \Delta t_{IR})^2 u^2(\alpha_{IR}) + (IR_t \alpha_{IR})^2 u^2(\Delta t_{IR}) + u^2(G_{20})$$

3.0 MEASURED RESULTS

The results tabulated below are based on the assumptions that measurements were taken at temperature of 20.5 °C and the instrument has a coefficient of thermal expansion of 11x10⁻⁶ / °C.

Nominal Length (mm)	Gauge Block Value, G ₂₀ (mm)	Instrument Reading, IR _t (mm)				Error, E ₂₀ (mm)	Standard Deviation (µm)
		1 st Reading	2 nd Reading	3 rd Reading	Mean		
0.0	-	0.000	0.000	0.000	0.000	0.000	0.000
3.1	3.100 05	3.101	3.100	3.101	3.1007	+ 0.001	0.577
6.5	6.499 97	6.501	6.501	6.500	6.5007	+ 0.001	0.577
9.7	9.700 00	9.699	9.700	9.699	9.6993	- 0.001	0.577
12.5	12.500 04	12.501	12.501	12.501	12.5010	+ 0.001	0.000
15.8	15.800 03	15.800	15.801	15.800	15.8003	0.000	0.577
19	19.000 07	19.001	19.001	19.001	19.0010	+ 0.001	0.000
21.9	21.900 00	21.900	21.900	21.900	21.9000	0.000	0.000
25	25.000 07	25.001	25.001	25.000	25.0007	0.000	0.577

EXAMPLE 13: MICROMETER CALIBRATION

4.0 STANDARD UNCERTAINTY EVALUATION

4.1 Uncertainty of instrument reading, $u(IR_t)$

Generally, the highest standard deviation is used to estimate the standard uncertainty due to repeated measurements, $u(Rpt)$. From the three measurements taken, we obtain:

$$\begin{aligned}u(Rpt) &= \frac{0.577}{\sqrt{n}} && \text{where } n = 3 \\ &= \frac{0.577}{\sqrt{3}} \\ &= 0.333 \mu\text{m} && \text{with degrees of freedom } \nu(Rpt) = n - 1 = 2\end{aligned}$$

The resolution (0.001 mm or 1 μm) of the micrometer is considered to be full-width of rectangular distribution and its associated standard uncertainty, $u(Res)$ is given as:

$$\begin{aligned}u(Res) &= \frac{0.001}{\sqrt{12}} \\ &= 0.289 \mu\text{m} && \text{with degrees of freedom } \nu(Res) = \infty\end{aligned}$$

Therefore,

$$\begin{aligned}u^2(IR_t) &= u^2(Rpt) + u^2(Res) \\ &= 0.333^2 + 0.289^2 \\ u(IR_t) &= 0.441 \mu\text{m}\end{aligned}$$

$$\begin{aligned}\nu(IR_t) &= \frac{u^4(IR_t)}{\frac{u^4(Rpt)}{\nu(Rpt)} + \frac{u^4(Res)}{\nu(Res)}} \\ &= \frac{0.441^4}{\frac{0.333^4}{2} + \frac{0.289^4}{\infty}} \\ &= 6.2 \\ &\approx 6\end{aligned}$$

4.2 Uncertainty of gauge block, $u(G_{20})$

Assuming that the uncertainty given in the calibration report with coverage factor $k = 2$ for gauge block size up to 25 mm is 0.10 μm , we obtain:

$$\begin{aligned}u(G_{20}) &= \frac{0.10}{2} \\ &= 0.05 \mu\text{m} && \text{with degrees of freedom } \nu(G_{20}) = \infty\end{aligned}$$

EXAMPLE 13: MICROMETER CALIBRATION

4.3 Uncertainty of coefficient of thermal expansion of the instrument, $u(\alpha_{IR})$

Given that the uncertainty of the coefficient of thermal expansion is represented by a rectangular distribution with boundaries of $\pm 2 \times 10^{-6} / ^\circ\text{C}$, we obtain:

$$\begin{aligned}u(\alpha_{IR}) &= \frac{2 \times 10^{-6} / ^\circ\text{C}}{\sqrt{3}} \\ &= 1.2 \times 10^{-6} / ^\circ\text{C} \quad \text{with degrees of freedom } \nu(\alpha_{IR}) = \infty\end{aligned}$$

4.4 Uncertainty of temperature deviation of instrument from 20°C , $u(\Delta t_{IR})$

Given that the temperature was measured as 20.5°C with an uncertainty represented by a rectangular distribution with boundaries of $\pm 0.5^\circ\text{C}$, we obtain:

$$\begin{aligned}u(\Delta t_{IR}) &= \frac{0.5}{\sqrt{3}} \\ &= 0.289^\circ\text{C} \quad \text{with degrees of freedom } \nu(\Delta t_{IR}) = \infty\end{aligned}$$

5.0 SENSITIVITY COEFFICIENTS

$$\begin{aligned}\frac{\partial E_{20}}{\partial IR_t} &= 1 - \alpha_{IR} \Delta t_{IR} \\ &= 1 - 11 \times 10^{-6} \times (20.5 - 20) \\ &= 0.99999\end{aligned}$$

Taken into consideration the maximum size of measurement,

$$\begin{aligned}\frac{\partial E_{20}}{\partial \alpha_{IR}} &= -IR_t \Delta t_{IR} \\ &= -25.0007 \times (20.5 - 20) \\ &= -12.5004 \text{ mm}^\circ\text{C}\end{aligned}$$

$$\begin{aligned}\frac{\partial E_{20}}{\partial \Delta t_{IR}} &= -IR_t \alpha_{IR} \\ &= -25.0007 \times 11 \times 10^{-6} \\ &= -0.00028 \text{ mm}/^\circ\text{C}\end{aligned}$$

$$\frac{\partial E_{20}}{\partial G_{20}} = -1$$

EXAMPLE 13: MICROMETER CALIBRATION

6.0 UNCERTAINTY BUDGET TABLE

Source of Uncertainty	Symbol x_i	Type	Uncertainty value	Probability distribution	Coverage factor	Standard uncertainty $u(x_i)$	Sensitivity coefficient c_i	$ c_i \times u(x_i)$ mm	Degrees of freedom ν
Repeatability of reading	$u(R_{pt})$	A	-	-	-	0.333 μm	1	0.000 333	2
Resolution of instrument	$u(Res)$	B	0.001 mm	Rectangular	$\sqrt{12}$	0.289 μm	1	0.000 289	∞
Gauge block	$u(G_{20})$	B	0.10 μm	Normal	2	0.05 μm	- 1	0.000 05	∞
Coefficient of thermal expansion of instrument	$u(\alpha_{IR})$	B	$\pm 2 \times 10^{-6} / ^\circ\text{C}$	Rectangular	$\sqrt{3}$	$1.2 \times 10^{-6} / ^\circ\text{C}$	- 12.5004 mm $^\circ\text{C}$	1.5×10^{-5}	∞
Temperature deviation of instrument	$u(\Delta t_{IR})$	B	$\pm 0.5 ^\circ\text{C}$	Rectangular	$\sqrt{3}$	0.289 $^\circ\text{C}$	- 0.000 28 mm / $^\circ\text{C}$	8.1×10^{-5}	∞

7.0 COMBINED STANDARD UNCERTAINTY, $u(E_{20})$

$$\begin{aligned}
 u_c^2(E_{20}) &= (1 - \alpha_{IR} \Delta t_{IR})^2 u^2(IR_t) + (IR_t \Delta t_{IR})^2 u^2(\alpha_{IR}) + (IR_t \alpha_{IR})^2 u^2(\Delta t_{IR}) + u^2(G_{20}) \\
 &= (0.99999)^2 (0.441)^2 + (12.5004)^2 \times (1.2 \times 10^{-6})^2 + (0.00028)^2 \times (0.289)^2 + (0.05)^2 \\
 &= 0.19698 \\
 u_c(E_{20}) &= 0.444 \mu\text{m}
 \end{aligned}$$

Effective degrees of freedom is thus:

$$\begin{aligned}
 \nu_{eff}(E_{20}) &= \frac{u_c^4(E_{20})}{\frac{(1 - \alpha_{IR} \Delta t_{IR})^4 u^4(IR_t)}{\nu(IR_t)} + \frac{(IR_t \Delta t_{IR})^4 u^4(\alpha_{IR})}{\nu(\alpha_{IR})} + \frac{(IR_t \alpha_{IR})^4 u^4(\Delta t_{IR})}{\nu(\Delta t_{IR})} + \frac{u^4(G_{20})}{\nu(G_{20})}} \\
 &= \frac{0.444^4}{\frac{0.99999^4 \times 0.441^4}{6} + 0 + 0 + 0} \\
 &= 6.2 \\
 &\approx 6
 \end{aligned}$$

8.0 EXPANDED UNCERTAINTY

From t-distribution table, at $\nu_{eff}(E_{20}) = 6$ and at an approximately 95 % level of confidence, $k = 2.52$.

Therefore,

$$\begin{aligned}
 U &= 2.52 \times 0.444 \\
 &= 1.119 \mu\text{m}
 \end{aligned}$$

EXAMPLE 13: MICROMETER CALIBRATION

9.0 UNCERTAINTY OF THE RANGE OF ERROR OF TRANSVERSE OF MICROMETER SCREW

The range of error of transverse of micrometer screw is given as:

$$(E_{20})_{\max} - (E_{20})_{\min} = 0.002 \text{ mm}$$

Hence,

$$\begin{aligned} u(\text{range of error}) &= \sqrt{2} \times u_c(E_{20}) \\ &= 0.628 \text{ } \mu\text{m} \end{aligned}$$

with degrees of freedom,

$$\begin{aligned} \nu(\text{range of error}) &= \frac{0.628^4}{\frac{0.444^4}{6} + \frac{0.444^4}{6}} \\ &= 12 \end{aligned}$$

From t-distribution table, $\nu(\text{range of error}) = 12$ and at an approximately 95 % level of confidence, $k = 2.23$. Therefore,

$$\begin{aligned} U &= 2.23 \times 0.628 \\ &= 1.400 \text{ } \mu\text{m} \\ \text{say,} \quad &= 0.002 \text{ mm} \end{aligned}$$

10.0 REPORTING OF RESULTS

The range of error of transverse of micrometer screw was found to be 0.002 mm. The associated expanded uncertainty of measurement is ± 0.002 mm, estimated at a level of confidence of approximately 95 % with coverage factor $k = 2.23$.

EXAMPLE 14 : ILLUMINATION METER CALIBRATION

1.0 INTRODUCTION

A schematic diagram of the calibration set-up is shown in Fig.1. A tungsten lamp of (calibrated) luminous intensity value is used as the standard, and the test meter is positioned at a distance away from the lamp. Calibration is done by comparing the reading of the test meter with the illuminance value determined by the inverse-square law from the luminous intensity of the lamp.

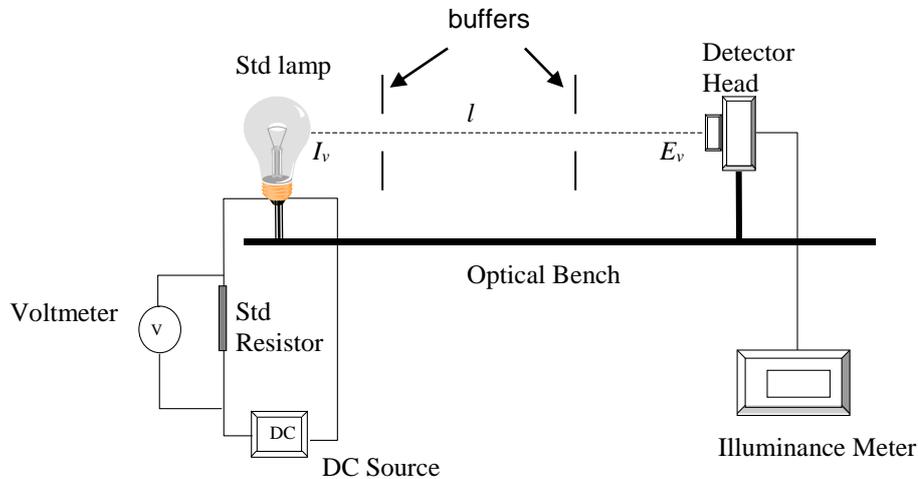


Figure 1: Schematic diagram of the calibration set-up

2.0 MATHEMATICAL MODEL

The luminous-intensity standard lamp generates an illuminance of known value at the front surface of the detector head of the test meter positioned in front of the lamp with a reading E_v :

$$E_v = I_v \frac{\cos \theta}{l^2} + \text{scattered light} + \Delta E_v$$

where

I_v : luminous intensity of the standard lamp run at a given current at which the correlated colour temperature of the lamp is approximately 2856K.

l : distance between the lamp filament and the front surface of the test meter.

$I_v \frac{\cos \theta}{l^2}$: the illuminance produced by the lamp according to the inverse-square law.

θ : the angle of deviation from normal incidence at the detector head.

ΔE_v : the error of the test meter; scattered light refers to input signal to the detector from light scattered or reflected from the wall, curtains or other surfaces in the optical set-up other than that from direct illumination by the lamp. Normally, the second and third terms are much smaller than the first one.

For simplicity, we will assume in the following discussions that

- the calibration is conducted in a dark room and sufficient measure is taken during calibration so that the contribution of scattered light is negligible;
- both the lamp and the detector head of the test meter are well aligned to a common optical axis determined by the optical bench so that $\cos \theta = 1$.

EXAMPLE 14: ILLUMINATION METER CALIBRATION

The measurement equation can then be simplified as

$$E_v = \frac{I_v}{I^2} + \Delta E_v \quad \dots(1)$$

For incandescent tungsten lamps, the luminous intensity varies with electrical current flowing through it according to

$$I_v = I_v(0) \left(\frac{i}{i_0} \right)^{6.25} \quad \dots(2)$$

where $I_v(0)$ is the reported (calibrated) luminous intensity value of the lamp under a current i_0 . The current (i) of the lamp is determined by measuring the voltage drop (V) across a standard resistor (resistance value R) that is connected in series with the lamp:

$$i = \frac{V}{R}$$

and

$$R = R_{20} [1 + \alpha(T - 20) + \beta(T - 20)^2] \quad \dots(3)$$

where R_{20} is the resistance of the standard resistor at 20 °C, α and β are the temperature coefficients of the resistor, and T is the resistor temperature in Celsius.

3. UNCERTAINTY EQUATION

The main sources of measurement uncertainty can be modelled from eq (1) through partial differentiation:

$$\left(\frac{dE_v}{E_v} \right)^2 = \left(\frac{dI_v}{I_v} \right)^2 + \left[2 \left(\frac{dl}{l} \right) \right]^2 + \left(\frac{d(\Delta E_v)}{E_v} \right)^2 \quad \dots(4)$$

where the last term represents the standard uncertainty of the error and the assumption $\Delta E_v \ll \frac{I_v}{I^2}$ is used in the derivation.

According to eq (2)

$$\left(\frac{dI_v}{I_v} \right)^2 = \left(6.25 \times \frac{di}{i} \right)^2 + \left(\frac{dI_v(0)}{I_v(0)} \right)^2 \quad \dots(5)$$

The second term represents the standard uncertainty (type B) of the lamp calibration. The first term is the standard uncertainty of current measurement determined by

$$\left(\frac{di}{i} \right)^2 = \left(\frac{dV}{V} \right)^2 + \left(\frac{dR}{R} \right)^2 \quad \dots(6)$$

EXAMPLE 14: ILLUMINATION METER CALIBRATION

and

$$\left(\frac{dV}{V}\right)^2 = (u(V_A))^2 + (u(V_B))^2 \quad \dots(7)$$

$$\begin{aligned} \frac{dR}{R} &= \frac{dR_{20}}{R_{20}} + \frac{[\alpha + 2\beta(T - 20)] \times \partial T}{1 + \alpha(T - 20) + \beta(T - 20)^2} \\ &\approx \frac{dR_{20}}{R_{20}} + [\alpha + 2\beta(T - 20)] \times \partial T \end{aligned} \quad \dots(8)$$

where the first and second terms in eq (7) represent the random (type A) uncertainty due to measurement repeatability and systematic (type B) uncertainties due to calibration of the volt meter respectively.

Substituting eqs (5) - (8) into eq (4), we have

$$\left(\frac{dE_v}{E_v}\right)^2 = \left(2\frac{dl}{l}\right)^2 + 6.25^2 \left[\left(\frac{dV}{V}\right)^2 + \left(\frac{dR}{R}\right)^2 \right] + \left(\frac{dI_v(0)}{I_v(0)}\right)^2 + \left(\frac{d(\Delta E_v)}{E_v}\right)^2$$

or

$$\left(\frac{dE_v}{E_v}\right)^2 = (2 \times u(l))^2 + (6.25 \times u(V_A))^2 + (6.25 \times u(V_B))^2 + (6.25 \times u(R))^2 + u(I_v)^2 + u(\Delta E_v)^2 \quad \dots(9)$$

where

$u(l)$: standard uncertainty of distance measurement.

$u(V_A)$: type A standard uncertainty of voltage measurement (voltmeter repeatability).

$u(V_B)$: type B standard uncertainty of voltage measurement due to voltmeter accuracy specification.

$u(R)$: standard uncertainty of resistance value of the standard resistor.

$u(I_v)$: standard uncertainty of standard lamp calibration.

$u(\Delta E_v)$: standard uncertainty of the error of illuminance meter measurement.

4.0 NUMERICAL EXAMPLE

4.1. $u(l)$, uncertainty of distance measurement

It can be determined using the accuracy of the scale on the bench and repeatability of the distance measurement. From past data, it is typically ± 1 mm at a distance 1 m or ± 0.1 % and can be considered as a type B uncertainty with a rectangular distribution. The relative standard uncertainty is therefore

$$\begin{aligned} u(l) &= \frac{0.1 \text{ \%}}{\sqrt{3}} \\ &= 0.0577 \text{ \%} \end{aligned}$$

The degrees of freedom is ∞ .

EXAMPLE 14: ILLUMINATION METER CALIBRATION

- 4.2. $u(V_A)$, type A standard uncertainty of voltage measurement (voltmeter repeatability), is determined by the standard deviation of the test data shown in the following table:

Data (mV)	36.6209
	36.6210
	36.6209
	36.6209
	36.6210
Mean (mV)	36.62094
STD (mV)	0.000055
STD of Mean (mV)	0.000024
Standard uncertainty (relative STD of Mean)	6.6E-5 %
Number of measurement (n)	5
Degrees of Freedom (n-1)	4

Note: STD = standard deviation.

- 4.3. $u(V_B)$, type B standard uncertainty of voltage measurement due to voltmeter accuracy specification.

For the range of concern (100 mV), it is given by the manufacturer as $\pm (0.0015 \% \text{ of reading} + 0.0003 \% \text{ of range})$ under the environmental condition $23 \text{ }^\circ\text{C} \pm 1 \text{ }^\circ\text{C}$ within 24 Hours. Using the data above, we have

$$\begin{aligned} u(V_B) &= 0.0015 \% \times 36.6 + 0.0003 \% \times 100 \\ &= 0.00085 \text{ mV} \\ &\Rightarrow 0.0023 \% \end{aligned}$$

with a degrees of freedom of ∞ .

- 4.4. $u(R)$, relative standard uncertainty of resistance value of the standard resistor, is determined by eq (8).

In this example, the values given by the manufacturer are $\alpha = 2.5 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$ and $\beta = 0.55 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$. The DC resistance of the standard resistor is given in it's calibration report as $0.009\,999\,26 \text{ } \Omega \pm 0.000\,000\,05 \text{ } \Omega$, temperature corrected to $23 \text{ }^\circ\text{C}$, at a level of confidence of approximately 95 % with a coverage factor $k = 2$. This corresponds to a relative standard uncertainty $\pm 0.000\,25 \%$, applicable also to dR_{20}/R_{20} . Assuming the temperature variation (dT) of the resistor during calibration is $\pm 1 \text{ }^\circ\text{C}$ under typical calibration lab environmental conditions $T = 23 \pm 1 \text{ }^\circ\text{C}$, we have

$$\begin{aligned} u(R) &= \sqrt{\left(\frac{dR_{20}}{R_{20}}\right)^2 + (\alpha + 2\beta(T - 20))^2 \delta T^2} \\ &= 0.00063 \% \end{aligned}$$

with a degrees of freedom of ∞ .

EXAMPLE 14: ILLUMINATION METER CALIBRATION

4.5 $u(I_v)$ - relative standard uncertainty of standard lamp calibration

An expanded uncertainty of $\pm 1.1\%$ is given in the calibration report at a confidence level approximately 95 % with a coverage factor of 2. This corresponds to a relative standard uncertainty $\pm 0.55\%$ with a degrees of freedom of ∞ .

4.6 $u(\Delta E_v)$ - standard uncertainty of the error of illuminance meter measurement (repeatability)

This is determined by measurement data given below

	Applied value ($I_v(0)/l^2$)	Measured value (E_v)	Error ($\Delta E_v = E_v - I_v(0)/l^2$)
data (lux)	1599.6	1559.2	- 40.4
		1560.2	- 39.4
		1560.2	- 39.4
		1559.2	- 40.4
		1559.2	- 40.4
Mean		1559.6	- 40.0
STD			0.548
STD of the mean:			0.245
Standard uncertainty (relative STD of the mean)		0.016 %	
Number of measurement		5	
Degrees of freedom		4	

Notes: From the definition given in eq (9), $u(\Delta E_v) = d(\Delta E_v)/E_v$. Since the applied value in the calibration is a constant, the standard deviations of the error $d(\Delta E_v)$ and of the test meter reading (dE_v) are identical. Therefore, $u(\Delta E_v)$ is equal to the relative standard deviation of the mean of the test meter readings.

5.0 UNCERTAINTY BUDGET TABLE

Source of Uncertainty	Symbol x_i	Type	Uncertainty value (%)	Probability distribution	Coverage factor	Standard uncertainty (%) $u(x_i)$	Sensitivity coefficient c_i	$ c_i \times u(x_i)$ (%)	Degrees of freedom ν_i
Unc of dist meas	$u(l)$	B	0.1	Rectangular	1.73	0.058	-2	0.12	∞
Repeatability of voltage meas	$u(V_A)$	A	0.000066	-	-	0.000066	6.25	0.00041	4
Accuracy of Voltmeter	$u(V_B)$	B	0.0023	Rectangular	1.73	0.0013	6.25	0.0084	∞
Cal unc of std resistor	$u(R)$	B	0.00063	Normal	1	0.00063	-6.25	0.0039	∞
Cal unc of std lamp	$u(I_v)$	B	1.1	Normal	2	0.55	1	0.55	∞
Repeatability of test meter reading	$u(\Delta E_v)$	A	0.016	-	-	0.016	1	0.016	5

EXAMPLE 14: ILLUMINATION METER CALIBRATION

6.0 COMBINED STANDARD UNCERTAINTY

The combined relative standard uncertainty $u_c(E_v)$ is given by eq (9), which can be rewritten as below

$$u_c^2(E_v) = \left(\frac{dE_v}{E_v} \right)^2 = (2 \times u(I))^2 + (6.25 \times u(V_A))^2 + (6.25 \times u(V_B))^2 + (6.25 \times u(R))^2 + u(I_v)^2 + u(\Delta E_v)^2$$

Substituting the data from Section 4, we have

$$u_c(y) = \left[\begin{aligned} &(2 \times 5.77 \times 10^{-2})^2 + (6.25 \times 6.6 \times 10^{-5})^2 + (6.25 \times 2.3 \times 10^{-3})^2 \\ &+ (6.25 \times 6.3 \times 10^{-4})^2 + (5.5 \times 10^{-1})^2 + (1.6 \times 10^{-2})^2 \end{aligned} \right]^{1/2} \% = 0.56 \%$$

7.0 EFFECTIVE DEGREES OF FREEDOM

It can be calculated from the general formulae given in the Guide and is essentially infinity in this example.

$$\begin{aligned} v_{eff} &= \frac{u_c^4(E_v)}{\sum_{i=1}^n \frac{c_i^4 u^4(x_i)}{v_i}} \\ &= u_c^4(y) \left[\frac{c_1^4 u^4(I)}{v_1} + \frac{c_2^4 u^4(V_A)}{v_2} + \frac{c_3^4 u^4(V_B)}{v_3} + \frac{c_4^4 u^4(R)}{v_4} + \frac{c_5^4 u^4(I_v)}{v_5} + \frac{c_6^4 u^4(\Delta E_v)}{v_6} \right]^{-1} \\ &= 0.56^4 \left[\frac{c_1^4 u^4(I)}{\infty} + \frac{(6.25 \times 6.6 \times 10^{-5})^4}{4} + \frac{c_3^4 u^4(V_B)}{\infty} + \frac{c_4^4 u^4(R)}{\infty} + \frac{c_5^4 u^4(I_v)}{\infty} + \frac{(1.6 \times 10^{-2})^4}{4} \right]^{-1} \\ &= 6.0 \times 10^6 \\ &= \infty \end{aligned}$$

8.0 EXPANDED UNCERTAINTY

For effective degrees of freedom $v_{eff} = \infty$, $k = 2$ is used at approximately 95 % level of confidence. Therefore, the expanded uncertainty

$$\begin{aligned} U &= k u_c(E_v) \\ &= 2 \times 0.56 \% \\ &= 1.1 \% \end{aligned}$$

9.0 REPORTING OF RESULTS

Description	Applied value (lux)	Indicated value (lux)	Deviation (%) (lux)
Range: Auto	1599.6	1559.6	-40 (-2.3%)

The expanded measurement uncertainty is ± 1.1 %, estimated at a level of confidence of approximately 95 % with a coverage factor $k = 2$.